UC Berkeley
Department of Electrical Engineering and Computer Sciences

EE126: Probability and Random Processes

Discussion Section 13
Fall 2018

Problem 1. **Conditioning on the Minimum of Uniforms**
If X and Y are independent $Uniform[0,1]$, show that

$$\mathbb{E}(Y \mid \min\{X,Y\}) = \frac{1}{4} + \frac{3}{4}\min\{X,Y\}.$$

Problem 2. **MMSE with Balls in Bins**
We throw $n \geq 1$ balls into $m \geq 2$ bins. Let X and Y represent the number of balls that land in bin 1 and 2 respectively.

1. Calculate $\mathbb{E}[Y \mid X]$.
2. What are $L[Y \mid X]$ and $Q[Y \mid X]$ (where $Q[Y \mid X]$ is the best quadratic estimator of Y given X)?

   Hint: Your justification should be no more than two or three sentences, no calculations necessary! Think carefully about the meaning of the MMSE.

Problem 3. **MMSE for Jointly Gaussian**
Let $\begin{bmatrix} X & Y & Z \end{bmatrix}^T \sim \mathcal{N}(\mu, \Sigma)$, and

$$\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 9 & 3 \\ 1 & 3 & 2 \end{bmatrix}.$$

Find $\mathbb{E}[X \mid Y, Z]$.