Problem 1. Ant
An ant is walking on the non-negative integers. At each step, the ant moves forward one step with probability \( p \in (0,1) \), or slides back down to 0 with probability \( 1-p \). What is the average time it takes for the ant to get to \( n \), where \( n \) is a positive integer, starting from state 0?

Problem 2. Before Absorption
Consider the Markov chain in Figure ??

Problem 3. Entropy of a Sum
Let \( X_1, X_2 \) be i.i.d. Bernoulli(1/2) (fair coin flips). Calculate \( H(X_1 + X_2) \) and show that \( H(X_1 + X_2) \geq H(X_1) \). In fact it is generally true that adding independent random variables increases entropy.

Note: It is known that the Gaussian distribution maximizes entropy given a constraint on the variance. Therefore, one intuitive interpretation of the CLT is that convolving independent random variables tends to increase uncertainty until the sum approaches the distribution which “maximizes uncertainty”, the Gaussian distribution. Proving the CLT along these lines is far from easy, however.