1. Running Sum of a Markov Chain

Let \((X_n)_{n \in \mathbb{N}}\) be a Markov chain with two states, \(-1\) and \(1\), and transition probabilities \(P(-1, 1) = P(1, -1) = a\) for \(a \in (0, 1)\). Define

\[Y_n = X_0 + X_1 + \cdots + X_n.\]

For what values of \(a\) is \((Y_n)_{n \in \mathbb{N}}\) a Markov chain?

2. Random Walk on an Undirected Graph

Consider a random walk on an undirected connected finite graph (that is, define a Markov chain where the state space is the set of vertices of the graph, and at each time step, transition to a vertex chosen uniformly at random out of the neighborhood of the current vertex). What is the stationary distribution?

3. Hitting Time with Coins

Let’s consider a sequence of fair coin flips.

(a) What’s the expected number of coin flips until we first see two heads in a row?

(b) What’s the expected number of coin flips until we see a head and then immediately a tail?