1. **Midterm**
   Solve all of the problems on the midterm again (including the ones you got correct). The midterm will be released by Thursday night (09/26) on the website.

2. **Really Random Binomial**
   You have a binomial random variable $X \sim \text{Bin}(n,u)$. Unfortunately, you lost information about what the value $u$ is, so you assume that $u$ is now a random variable $U \sim \text{Unif}[0,1]$, since you know it is a probability. Given that you sample from this binomial distribution and observe $k$ successes, find the posterior distribution of $U$.

   **Hint:** Use MGFs to compute $P(X = k)$ instead of integrating the distribution directly. The binomial theorem might also be useful here. Finally, recall the identity $\sum_{i=0}^{n} s^{i} = \frac{1-s^{n+1}}{(1-s)}$.

3. **Poisson Bounds**
   Let $X$ be the sum of 20 i.i.d. Poisson random variables $X_1, \ldots, X_{20}$ with $E[X_1] = 1$. Use the following techniques to upper bound $P(X \geq 26)$.
   
   (a) Markov’s Inequality
   (b) Chebyshev’s Inequality
   (c) Chernoff Bound

4. **Tricky Markov Bound**
   Suppose $E[X] = 0$, $\text{var}(X) = \sigma^2 < \infty$, and $\alpha > 0$. Prove the following bound:
   
   $P(X \geq \alpha) \leq \frac{\sigma^2}{\alpha^2 + \sigma^2}$.

5. **Confidence Interval Comparisons**
   In order to estimate the probability of a head in a coin flip, $p$, you flip a coin $n$ times, where $n$ is a positive integer, and count the number of heads, $S_n$. You use the estimator $\hat{p} = S_n/n$.

   (a) You choose the sample size $n$ to have a guarantee
   
   $P(|\hat{p} - p| \geq \epsilon) \leq \delta$.
   
   Using Chebyshev Inequality, determine $n$ with the following parameters. Note that you should not have $p$ in your final answer.
(i) Compare the value of \( n \) when \( \epsilon = 0.05, \delta = 0.1 \) to the value of \( n \) when \( \epsilon = 0.1, \delta = 0.1 \).

(ii) Compare the value of \( n \) when \( \epsilon = 0.1, \delta = 0.05 \) to the value of \( n \) when \( \epsilon = 0.1, \delta = 0.1 \).

(b) Now, we change the scenario slightly. You know that \( p \in (0.4, 0.6) \) and would now like to determine the smallest \( n \) such that

\[
P\left(\left|\frac{\hat{p} - p}{p}\right| \leq 0.05\right) \geq 0.95.
\]

Use the CLT to find the value of \( n \) that you should use. Recall that the CLT states that the sum of IID random variables tends to a normal distribution with the sample mean and variance as its parameters for \( n \) large enough.