Problem 1. Example 6.9 on textbook: Queuing system.
Packets arrive at a router in Internet, where they are stored in a buffer and then transmitted. The storage capacity of the buffer is m: if m packets are already present, any newly arriving packets are discarded. We discretize time in very small periods, and we assume that in each period, at most one event can happen that can change the number of packets stored in the router, i.e. an arrival of a new packet or a completion of the transmission of an existing packet. In particular, we assume that at each period, exactly one of the following occurs: a new packet arrives with probability λ; one existing packet completes transmission, with probability μ if there are packets in the buffer.
a) Conceptual question: how many steps to fully define a Markov chain? Why and when a process can be modeled as Markov chain?
b) What is an appropriate Markov chain model for this problem? Does this Markov chain have steady state distribution?
c) What is the average queue length in steady state?
d) What is the probability of dropping packet? If we want this probability to be less than 0.01, how large the buffer should be?

Problem 2.
A spider is hunting a fly moves between location 1 and 2 according to a Markov chain with transition probability matrix P_1, starting at location 1. The fly, unaware of the spider, starts in location 2 and moves according to a Markov chain with transition matrix P_2. The spider catches the fly and the hunt ends whenever they meet in the same location.

$$P_1 = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.4 & 0.6 \\ 0.6 & 0.4 \end{bmatrix}.$$

a) Can you provide a Markov chain model for this process? Identify the transient and recurrent states.
b) Find the probability that at time n the spider and the fly are both at their initial location.
c) What is the average duration of the hunt?

Problem 3. Frog escaping from a well.- *please do it by yourselves. Hint: apply the same idea we used in problem 2.c.*
A frog someday find himself in the bottom of a well with a three steps stair towards the top to freedom. He decides to escape from the well. However, he is lazy and memoryless. He wakes up everyday to jump up or down one step with equal probability and then goes to sleep again, regardless what he did in previous days.
a) What is the corresponding Markov chain model for this problem?
b) What is the average number of days before he escapes to freedom?