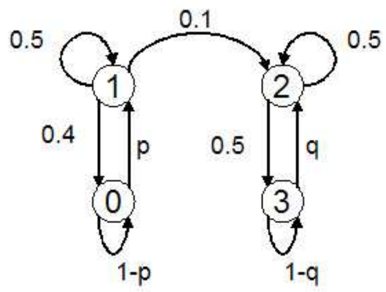
Problem 1.
A Markov chain is shown below.



a) Identify the transition states and recurrent states.
b) Now we want to estimate p and q. We start at state '0', run the chain and observe its evolution until time $n = 10000$. Can you come out with a scheme to estimate p and q? Is your scheme optimal? Why? (hint: it is similar to the polling problem in homework, where we try to estimate the fraction of people that vote for Kerry.)
c) How you evaluate the reliability of your estimates?
d) As $n \to \infty$, will your estimate of q become more and more reliable? How about the estimate of p?

Problem 2. Hidden Markov chain and more.
Assume whether a router is congested in current time slot depends only on its status in last time slot. If it was congested last time, then it is congested with probability 0.9 in current time slot; otherwise it is congested with probability 0.1.
a) Define a model for this 'congested'-'free' process. Is it a Markov chain?
b) Suppose a sender send one packet per time slot to the router. If the router is congested, it drops the packet; otherwise it drops the packet with probability p and serve it with probability $1 - p$. Here we assume $p = 0.5$. Define a model for the process of packet loss. Is it a Markov chain?
c) Now you, the receiver, observe packet loss event and try to infer whether the router is congested. In practice, you observe one packet loss events in consecutive time slots, i.e. no packet loss in time slot 1 and a packet loss event in time slot 2; you want to detect the router's status in time slot 1 and 2. If you make MAP detections independently based on each individual observation in each time slot, what is the error probability? If you make joint MAP detection decisions based on both observations in two time slots, what is the error probability? Does that meet your intuition?