Name: ______________________ Student ID No: ____________

UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Ren Fall 1997

# EECS 126 — MIDTERM #2

**November 17, 1997, Monday 7-9 p.m.**

**[42 pts.] 1.** Given the joint probability density of two RVs $X$ and $Y$

$$f_{XY}(x, y) = \begin{cases} k(x+y) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

**a)** Find the value of $k$, and the cdf $F_{XY}(x, y)$. (6 pts.)

**b)** Find $F_X(x), F_Y(y), f_X(x), f_Y(y)$. (6 pts.)

**c)** Find the probability that $|X - Y| \le 1/2$. (6 pts.)

**d)** Find $f_{X|Y}(x|y)$. (6 pts.)

**e)** Find the minimum mean square error estimator of $X$ given $Y$. Compute the resulting mean square error. (6 pts.)

**f)** Find the linear minimum mean square error estimator of $X$ given $Y$. Compute the resulting mean square error. (6 pts.)

**g)** Are $X$ and $Y$ independent? Uncorrelated? Orthogonal? Explain your answer. (6 pts.)

**[35 pts.] 2.** An electronic system has $n$ components. Let the lifetime of each component be $X_i,\ i = 1, 2, \ldots, n$, in hours. Assume that $X_i,\ i = 1, 2, \ldots, n$, are mutually independent, and have identical density $f_{X_i}(x) = e^{-x}, \quad x \ge 0$. Let the lifetime of the system be $Y$.

**a)** Suppose the system works only if all $n$ components work. Find the pdf and expectation of $Y$. (10 pts.)

**b)** Suppose we already know that the system has already lasted 10 hours. Find the conditional pdf and expectation of $Y$. (12 pts.)

**c)** To increase reliability, we use redundancy by increasing the number of components from $n$ to $2n$. Suppose the system works so long as there are at least $n$ components working. Find the cdf of $Y$. (13 pts.)

**[23 pts.] 3.** Let $X_1, X_2, \ldots$ be a sequence of i.i.d. RVs with mean $\mu$ and unit variance. Suppose $\mu$ is unknown.

**a)** Propose a scheme to estimate $\mu$ from $X_1, \ldots, X_n$. (5 pts.)

**b)** Suppose your estimate of $\mu$ based on $X_1, \ldots, X_n$ is denoted as $\hat{\mu}_n$. Using Central Limit Theorem, find a range of $n$ that would guarantee the quality of the estimate in the following sense:

$$P(|\hat{\mu}_n - \mu| \le 0.1) \ge 0.9. \text{ (18 pts.)}$$