[20 pts.] 1a. Suppose that $E, F$ are events and $P(E) = 0.4$. What can you say about $P(E|F)$ if:

i) $E$ and $F$ are independent?

ii) $E$ and $F$ are mutually exclusive?

iii) $F \subset E$?

iv) $E \subset F$?

[10 pts.] b. If the occurrence of event $B$ makes $A$ more likely (i.e., $P(A|B) > P(A)$), then does the occurrence of event $A$ make $B$ more likely? Justify your answer.

[30 pts.] 2. There are 2 machines having lifetimes distributed with cdf’s $F_1$ and $F_2$. Suppose one of the 2 machines is randomly picked with equal probability and put in operation at time 0. Conditional on the fact that the machine is still running at time $t$, what is the probability that it is machine 1 that was picked?

[20 pts.] 3a. Consider a binary channel with cross-over probability

$$
P(\text{output } = 1 | \text{input } = 0) = \varepsilon_1$$
$$
P(\text{output } = 0 | \text{input } = 1) = \varepsilon_2$$

Suppose $P(\text{input } = 0) = p \quad . \quad P(\text{input } = 1) = 1 - p$

Further suppose you use a detection rule which decides that 0 is transmitted if 0 is received, and 1 is transmitted if 1 is received. Find the probability that you will make an error.

[20 pts.] b. Suppose now that $p = \frac{1}{2}, \varepsilon_1 = \varepsilon_2 = \varepsilon < \frac{1}{2}$. A student thinks that a random detection rule can perform better than the detection rule above. Namely, the student flips a biased coin with $P(\text{head}) = \varepsilon$. If the coin lands on a tail, the student decides that what is transmitted is the same as what is received; if the coin lands on a head, he decides that what is transmitted is opposite to what is received. What is the probability that the student makes an error using this rule? Is this a better rule than the one in (a)?