EE 126: Probability and Random Processes

Problem Set 1

Due: January 29 in class

Problems 3, 8, 12 and 15 in http://www.athenasc.com/CH1-prob-supp.pdf. In each of the last three problems, the underlying probability model is not explicitly given so you have to define it. Give reasons why your model is a reasonable one.

5. Suppose A and B are two events with known probabilities $P(A)$ and $P(B)$.

a) Can you compute $P(A \cup B)$ in terms of $P(A)$ and $P(B)$? If so, explain. If not, find the largest and smallest possible values $P(A \cup B)$ can take in terms of $P(A)$ and $P(B)$ and give examples in which these values can be attained (i.e. give upper and lower bounds for $P(A \cup B)$.)

b) Repeat (a) with $P(A \cap B)$ instead of $P(A \cup B)$.

6. Let $\Omega = [x_{\min}, x_{\max}]^2$ be the sample space containing outcomes (x_1, x_2) of two consecutive speech signal values.

a) What is a natural probability law for modelling the situation where the values are completely random and have nothing to do with each other?

b) What would the probability law look like if there is a strong dependence between the two values such that when one is large then it is likely that the other is large, etc.? Sketch a representative probability law.

c) What would the probability law look like if there is a strong dependence between the two values such that when one is large then it is likely that the other is small and vice versa? Sketch a representative probability law.

7. Can one define a uniform probability law on a countably infinite sample space? Explain. What property must *any* probability law on such a sample space satisfy? Explain

8. Let Ω_∞ be the sample space of all infinite sequences of coin flips.

a) Find a one-to-one correspondence between the the elements of $[0, 1]$ and Ω_∞.

b) Let A be the event that the first 5 flips are all heads. What is the corresponding event in $[0, 1]$?

c) The uniform probability law on $[0, 1]$ yields a natural probability law on Ω_∞ through the correspondence in part a). What is the probability of the event A under this law?

d) Let Ω_{finite} be the sample space of all finite sequences of coin flips. Can you find a natural correspondence (not necessarily one-to-one) between the elements of Ω_{finite} and $[0, 1]$?

e) Using parts a) and d), find a correspondence between the outcomes in Ω_{finite} and Ω_∞.