

EECS 126: Probability and Random Processes

Problem Set 10

Due: Thurs, April 29, 2004

1. Consider the two-state Markov chain discussed in class with transition probability from state 0 to 1 equal to p and from state 1 to 0 equal to q .

a) Simulate this Markov chain on MATLAB or your favorite program and plot the sample paths for different values of p and q , covering the various cases we discussed in lecture.

b) Fix $p = 0.1$, $q = 0.3$. Plot the state distribution as a function of time, starting from both the initial state 0 and the initial state 1. See how convergence to the steady-state distribution occurs.

2. Consider the buffered switch example discussed in class, but suppose now there are two independent Bernoulli arrival streams instead of one. For each stream, the probability a packet arrives at any time slot is p and the probability of zero arrival is $1 - p$, and the arrivals are independent over time and across streams. The service rate of the switch is one packet per second. The buffer can contain up to b packets.

a) Give a Markov chain model, specifying the state space and transition probabilities.

b) Which states are recurrent? Which are transient? Does the chain approach a unique steady-state distribution starting from any initial distribution?

c) If the answer to part (b) is yes, compute explicitly the steady-state distribution for $b = 5$ and for several different values of $p = 0.1, 0.4, 0.9$. (You can use MATLAB to help you with the computation.) For different ranges of the values for p , the steady-state distribution looks qualitatively different. Can you guess what these ranges are?

3. This question is for the same model in Q.2. We fix $p = 0.4$, $b = 5$.

a) Simulate the Markov chain and plot a sample path.

b) Simulate the chain and plot, as a function of time n , the fraction of time that the buffer is full up til time n . Do this for several different initial state. Do you observe any convergence? If so, does the limit depend on the initial state? Can you connect the limit to the steady-state distribution you calculated in Q. 2 c)?

c) Simulate the chain and plot, as a function of time n , the fraction of packets that are lost up til time n . (Packets get lost because the buffer is full when they arrive.) Do this for several different initial state. Do you observe any convergence? If so, does the limit depend on the initial state? Can you venture a guess on how this limit could have been computed from the steady-state distribution you calculated in Q. 2 c)?

4. a) In class we derived equations for the expected time to hit a given state. Write these equations in matrix form in terms of the transition probability matrix P .

b) Using (a), solve for the expected time until the buffer first fills starting with an empty buffer in the model in Q.2. (You can use MATLAB.) Do this calculation for $p =$

0.1, 0.4, 0.9. Qualitatively, how does the expected time depend on p ? Is this consistent with your intuition?

5. a) Consider the two-state Markov chain in Q. 1 with $p \neq 0$ and $q \neq 0$. Suppose now we don't know what the values of p and q are and we want to estimate them based on observing a sample path of the chain. We start at state 0, run the chain and observe its evolution until time n . Find the maximum likelihood estimate of p and q given your observations. What do you expect the accuracy of your estimates to be as $n \rightarrow \infty$? (Just an intuitive justification to this last question is needed.)

(Hint: it may be easier to deal with the log of the probabilities rather than the probabilities themselves when you compute the ML estimate.)

b) Now suppose $q = 0$. We repeat the experiment in (a) and want to estimate p . Find the ML estimate of p given your observation up to time n ? What do you expect the accuracy of your estimate as $n \rightarrow \infty$? (Just an intuitive explanation to this last question is needed.)