EECS 126: Probability and Random Processes

Problem Set 11

Not to be handed in

1. (Maximum likelihood refresher).

(a) You observe a sequence of n independent Bernoulli trials each with success probability p, but you don't know p. Compute the maximum likelihood estimate of p

(b) You observe n i.i.d. exponential distributed random variables all with mean either μ_1 or μ_2, but you don't know which. Find the maximum likelihood estimate based on your observations.

2. A binary information sequence $X_0, X_1, \ldots$ of 0's and 1's is modelled by a two state Markov chain with transition probability p from 0 to 1 and q from 1 to 0, starting in steady state. The information sequence is sent through a binary symmetric channel with crossover probability ϵ and the output is the sequence $Y_0, Y_1, \ldots$.

(a) Suppose you observe the output sequence 1001. Let $\epsilon = 0.2$, $p = 0.4$, $q = 0.7$. Compute the MAP estimate of the input sequence given your observations by running the Viterbi algorithm by hand.

(b) For general values of the parameters of the problem and general observed sequence $Y_0 = y_0, Y_1 = y_1, \ldots, Y_n = y_n$, state as explicit as you can the algorithm to compute the MAP estimate.

(c) Now let's consider a different question. We observe $Y_0 = y_0, \ldots, Y_n = y_n$ and would like to *predict* the value of Y_{n+1}. Suppose $p = q = 0.03$ and $\epsilon = 0.3$. Come up with an intuitive rule to try to predict. (No need to be optimal.)

(d) Is the output sequence a Markov chain? Explain.

3. (channel identification) In many communication scenarios, one does not know what the communication channel is beforehand and would like to estimate it based on the observed output from the channel. We considered a simplified version here.

We send an input sequence $X_0, X_1, \ldots,$, i.i.d. equiprobable to be $+1$ or -1. The output sequence $Y_1, Y_2, \ldots$ is given by:

$$Y_i = h_0 X_i + h_1 X_{i-1} + W_i \qquad \text{i =1,2, } \ldots, \tag{1}$$

where W_i's is a sequence of i.i.d. Gaussian $N(0, \sigma^2)$ noise, independent of the input sequence. Thus, the channel is a FIR filter with two coefficients h_0 and h_1.

(a) For given values of h_0 and h_1, model the output as a Hidden Markov process. What is the underlying Markov chain? How many states does it have?

(b) Suppose we know that the channel is either $h_0 = 2, h_1 = 1$ or $h_0 = 4, h_1 = 0.5$, but we don't know which. Based on n observation of the output, find an algorithm to figure out which channel is more likely to yield the output.