Name: ________________________ Student ID No: _____________

UNIVERSITY OF CALIFORNIA

College of Engineering
Department of Electrical Engineering and
Computer Sciences

Professor Tse Fall 1999

EECS 126 — MIDTERM #1

23 September 1999

Please explain your reasoning carefully. You can leave answers in summation form if you don't see any simple way of summing them.

Some pmf's:

Binomial: $P_X(k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 0, \ldots, n$

Geometric: $P_X(k) = p(1-p)^k, \quad k = 0, 1, \ldots$

Poisson: $P_X(k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}, \quad k = 0, 1, \ldots$

[30 pts.] 1. True or false. If true, justify. If false, give a counter-example. No marks without justification.

a) A and B are independent events implies that A^c and B are independent. (5 pts.)

b) A and B are independent, and B and C independent implies A and C are independent. (5 pts.)

c) $P(A \cap B \cap C) = P(A|B)P(B|C)P(C)$, where A, B, C are events with non-zero probabilities. (5 pts.)

d) A and B independent implies A and B are conditionally independent given any event C. (A, B are said to be conditionally independent given C if $P(A \cap B|C) = P(A|C)P(B|C)$.) (5 pts.)

e) $P(A) = P(A|B^c) + P(A|B)$. (5 pts.)

f) $P(A|B) = P(A|B \cap D)P(D|B) + P(A|B \cap D^c)P(D^c|B)$. (5 pts.)

[20 pts.] 2. n information bits $X_1, X_2, \ldots, X_n$ are transmitted through a noisy communication channel. Each bit is independently corrupted by a binary symmetric channel with crossover probability ε. One extra parity check bit X_{n+1} is also transmitted

$$X_{n+1} = X_1 \oplus X_2 \oplus \ldots \oplus X_n$$

and it is also independently corrupted by a BSC with crossover probability ε. The receiver does a parity check on the output bits $Y_1, \ldots, Y_n, Y_{n+1}$ to determine if any errors occur in the n information bits, i.e., it checks if

$$Y_1 \oplus Y_2 \oplus \ldots \oplus Y_n \overset{?}{=} Y_{n+1}$$

Find the probability that there is at least 1 error in the information bits given that the parity check test is passed.

[25 pts.] 3. There are $2n$ users sharing a communication link of capacity k packets per time slot. Assume $k \geq n$. At a given time slot, a user may either have none or one packet to transmit. n of the users belong to class 1, each with probability p having a packet to transmit. The rest belong to class 2, each with probability q having a packet to transmit. All users behave independently. Packets get dropped when the total number of packets transmitted during a time slot exceeds the link capacity.

a) Find the expected total number of packets being transmitted in a time slot. (5 pts.)

b) Find the probability that at least one packet gets dropped. (10 pts.)

c) Suppose a scheduler always gives the highest priority to class 1 packets, i.e., all the packets from class 1 are transmitted before class 2 packets are transmitted. Find the probability that a class 1 packet is dropped. Find the probability that a class 2 packet is dropped. (Hint: Assumption $k \geq n$ is useful.) (10 pts.)

[25 pts.] 4. n people arrive at a party and leave their hats at the door. They leave one after another and each randomly picks a hat from the remaining ones.

a) Find the probability that the i^{th} person gets back his/her own hat. (5 pts.)

b) Find the expected number of people who get their own hat. (5 pts.)

c) Let E_i be the event that the i^{th} person gets back his/her own hat. Are the events E_i and E_j independent, for $i \neq j$? Justify your answer both by an explicit calculation and an intuitive explanation. (10 pts.)

d) Find the variance of the number of people who get back their own hat. (Hint: This part is more difficult. The calculation in (c) may be useful.) (5 pts.)