EECS 126 — Diagnostic Quiz

1. Let $A =$ set of all persons taller than 6 feet
   $B =$ set of all persons heavier than 150 lbs.
   $C =$ set of all persons who wear glasses

   a) What is the set of all persons who are shorter than 6 feet or wear glasses?

   b) What is the set of all persons who are taller than 6 feet and are heavier than 150 lbs., but do not wear glasses?

   c) Express these statements in terms of $A, B, C$:
      i) There is no one who wears glasses and is shorter than 6 feet.
      ii) If a person is lighter than 150 lbs., he/she must either wear glasses or is taller than 6 feet.

   d) Can you relate:
      i) $A \cup B$ and $A \cap B$
      ii) $A \cup B$ and $A^c \cap B^c$

   Draw Venn diagrams and also express your mathematical statements in terms of plain English.

   e) If $A \subseteq B$, what can you say about the relationship between $A^c$ and $B^c$? What is the corresponding statement in English?
2a) Compute: \( \sum_{i=1}^{n} p^i \) and \( \sum_{i=1}^{n} ip^i \)

b) Compute: \( \int_0^\infty e^{-ax} \, dx \) and \( \int_0^\infty e^{ax} \, dx \), \( (a > 0) \)

c) Compute: \( \int_0^1 xe^{-x} \, dx \)

d) Is it true that:

i) \( \int_0^1 f(x) + g(x) \, dx = \int_0^1 f(x) \, dx + \int_0^1 g(x) \, dx \) for all \( f, g \)?

ii) \( \int_0^1 \int_0^1 f(x)g(y) \, dx \, dy = \int_0^1 f(x) \, dx \int_0^1 g(x) \, dx \)?

e) Plot the curves:

i) \( f(x) = e^{\frac{-x^2}{2}}, \quad -\infty < x < \infty \)

ii) \( f(x) = \log x \)
f) Plot the curve:

\[ ax^2 + by^2 = c \]

where \( c > 0 \) and \( a > b \). Label the key features.

What about the curve:

\[ ax^2 + 2ax + by^2 - 2by = c \]