EECS 126: Probability and Random Processes

Problem Set 1

Due on Thursday, January 27th 2005 in class

Note: Please submit a photocopy of your work.

If you collaborate on the assignment, please list the names of students in your study group

Problem 5. We are given that $P(A^c) = 0.6$, $P(B) = 0.3$, and $P(A \cap B) = 0.2$. Determine $P(A \cup B)$.

Problem 9. Alice and Bob each choose at random a number between zero and two. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:

A: The magnitude of the difference of the two numbers is greater than 1/3.

B: At least one of the numbers is greater than 1/3.

C: The two numbers are equal.

D: Alice's number is greater than 1/3.

Find the probabilities $P(A)$, $P(B)$, $P(A \cap B)$, $P(C)$, $P(D)$, $P(A \cap D)$.

Problem 11. Show the following generalizations of the formula

$$P(A \cup B \cup C) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C).$$

(a) Let A, B, C, and D be events. Then

$$P(A \cup B \cup C \cup D) = P(A) + P(A^c \cap B) + P(A^c \cap B^c \cap C) + P(A^c \cap B^c \cap C^c \cap D).$$

(b) Let $A_1, A_2, \ldots, A_n$ be events. Then

$$P\left(\cup_{k=1}^{n} A_k\right) = P(A_1) + P(A_1^c \cap A_2) + P(A_1^c \cap A_2^c \cap A_3) + \cdots + P(A_1^c \cap \cdots \cap A_{n-1}^c \cap A_n).$$

Problem 19. A magnetic tape storing information in binary form has been corrupted, so it can only be read with bit errors. The probability that you correctly detect a 0 is 0.9, while the probability that you correctly detect a 1 is 0.85. Each digit is a 1 or a 0 with equal probability. Given that you read a 1, what is the probability that this is a correct reading?

Problem 22. A parking lot consists of a single row containing n parking spaces ($n \geq 2$). Mary arrives when all spaces are free. Tom is the next person to arrive. Each person makes an equally likely choice among all available spaces at the time of arrival. Describe the sample space. Obtain $P(A)$, the probability the parking spaces selected by Mary and Tom are at most 2 spaces apart.

Problem 25. A particular jury consists of 7 jurors. Each juror has a 0.2 chance of making the wrong decision, independently of the others. If the jury reaches a decision by majority rule, what is the probability that it will reach a wrong decision?

Problem 26. Three persons roll a fair n-sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12}, A_{13}, and A_{23} are pairwise independent but are not independent.

Problem 30. We are told that events A and B are independent. In addition, events A and C are independent. Is it true that A is independent of $B \cup C$? Provide a proof or counterexample to support your answer.