EECS 126: Probability and Random Processes
Problem Set 10
Due on Thursday, April 21th 2005 in class

Problem 1. We are given a coin for which the probability of heads is p $(0 < p < 1)$ and the probability of tails is $1 - p$. Consider a sequence of independent flips of the coin.

(a) Let Y be the number of flips up to and including the flip on which the first head occurs. Write down the PMF of Y.

(b) Let X be the number of heads that occur on any particular flip. Write down $\mathbf{E}[X]$ and $\text{var}(X)$.

(c) Let K be the number of heads that occur on the first n flips of the coin. Determine the PMF, mean, and variance of K.

(d) Given that a total of exactly six heads resulted from the first nine flips, what is the conditional probability that both the first and seventh flips were tails?

(e) Let H be the number of heads that occur on the first twenty flips. Let C be the event that a total of exactly ten heads resulted from the first eighteen flips. Find $\mathbf{E}[H \mid C]$ and the conditional variance $\text{var}(H \mid C)$.

Problem 3. **[D]** To cross a single lane of moving traffic, we require at least a duration d. Successive car interarrival times are independently and identically distributed with probability density function $f_T(t)$. If an interval between successive cars is longer than d, we say that the interval represents a single opportunity to cross the lane. Assume that car lengths are small relative to intercar spacing and that our experiment begins the instant after the zeroth car goes by. Determine, in as simple form as possible, expressions for the probability that:

(a) We can cross for the first time just before the nth car goes by.

(b) We shall have had exactly m opportunities by the instant the nth car goes by.

(c) The occurrence of the mth opportunity is immediately followed by the arrival of the nth car.

Problem 5. **[D]** Fred is giving out samples of dog food. He makes calls door to door, but he leaves a sample (one can) only on those calls for which the door is answered and a dog is in residence. On any call the probability of the door being answered is $3/4$, and the probability that any household has a dog is $2/3$. Assume that the events "Door answered" and "A dog lives here" are independent and also that the outcomes of all calls are independent.

(a) Determine the probability that Fred gives away his first sample on his third call.

(b) Given that he has given away exactly four samples on his first eight calls, determine the conditional probability that Fred will give away his fifth sample on his eleventh call.

(c) Determine the probability that he gives away his second sample on his fifth call.

(d) Given that he did not give away his second sample on his second call, determine the conditional probability that he will leave his second sample on his fifth call.

(e) We will say that Fred "needs a new supply" immediately **a**fter the call on which he gives away his last can. If he starts out with two cans, determine the probability that he completes at least five calls before he needs a new supply.

(f) If he starts out with exactly m cans, determine the expected value and variance of D_m, the number of homes with dogs which he passes up (because of no answer) before he needs a new supply.

Problem 8. A particular medical operation proves fatal in 1% of the cases. Find an approximation to the probability that there will be at least 2 fatalities in 200 operations.

Problem 10. A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

(a) If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.

(b) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.

(c) Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge.

Problem 13. A certain police officer stops cars for speeding. The number of red sports cars she stops in one hour is a Poisson process with rate 4, while the number of other cars she stops is a Poisson process with rate 1. Assume that these two processes are independent of each other. Find the probability that this police officer stops at least 2 ordinary cars before she stops 3 red sports cars.

Problem 18. Based on your understanding of the Poisson process, determine the numerical values of a and b in the following expression and explain your reasoning.

$$\int_t^\infty \frac{\lambda^6 \tau^5 e^{-\lambda\tau}}{5!}\, d\tau = \sum_{k=a}^{b} \frac{(\lambda t)^k e^{-\lambda t}}{k!}.$$