Problem 1.1 Markov Chain
Let \( X_n \) be a Bernoulli i.i.d. process with parameter \( p = \frac{1}{2} \).

a. Is \( X_n \) a Markov Chain? Why or why not?

b. Another sequence \( Y_n \) is defined by \( Y_n = X_n + X_{n-1} \). Is \( Y_n \) a Markov process? Why or why not?

c. Another sequence \( Z_n \) is defined by \( Z_n = (X_n, X_{n-1}) \). Is \( Z_n \) a Markov process? Why or why not?

Problem 1.2 Finite State Markov Chain
Bob goes to Las Vegas. He does not want to lose a lot of money so decides to gamble with only $3 and to stop playing if he loses these $3 dollars or reaches $5. He approaches to a roulette wheel, which contains 18 red, 18 black and 2 green holes. He decides to always bet $1 on red on the roulette wheel.

a. Formulate the Markov Chain corresponding to the amount of dollars that Bob has and classify the states.

b. Find the transition probability matrix of the Markov Chain. Determine its eigenvalues and eigenvectors. Express the row vector containing probability that Bob has $i at the end of \( n \)-th game for \( i = 0, 1, 2, 3, 4, 5 \) in terms of the eigenvalues and eigenvectors. (Use MATLAB to help yourself out)

c. Find the steady state probability that Bob has $i at the end of \( n \)-th game for \( i = 0, 1, 2, 3, 4, 5 \).

d. Repeat part (b) and (c) for the case when Bob starts with $2 and stops playing if he loses all his money or has $5.

Problem 1.3 Processing Jobs
Consider a single job processor with a queue. Suppose that time is slotted into durations \( \tau \) and jobs arrive at the beginning of the time slot as a Bernoulli process \( B_k \) with an independent probability \( \lambda \tau \) that a given time slot will contain a job. The jobs will accumulate in a first-come first serve queue and the currently active job had an independent probability \( q = \rho \tau \) of being completed in this time slot. If a job is completed at time \( k\tau \), then it is removed from the queue and sent on to the next processor and we can say that \( Y_k = 1 \). If no job is completed then \( Y_k = 0 \).

a. Give an explicit model for the above.
b. What are the steady state probabilities for the queue size? What is the condition required for steady state to even exist?

c. Suppose that $\rho$ and $\lambda$ are such that the queue is stable (ie. has a steady state distribution). Furthermore, suppose that the queue is in steady state. What is the expected time till the next job is completed?

d. Let $\tau$ go to zero. What does the process of jobs coming into the processor look like?

e. Let $\tau$ go to zero. What is the steady state distribution of the queue sizes?

f. Let $\tau$ go to zero and assume that the queue is in steady state. What is the expected time till the next job is completed?

g. Bonus: Let $\tau$ go to zero and assume that the queue is in steady state. What is the pdf for the time of the next job completion?