Spring 2005: EECS126 Homework 12

Due on Thursday May 5th, 2005 in class

Problem 1.1 *Markov Chain*

Let X_n be a Bernoulli i.i.d. process with parameter $p = \frac{1}{2}$.

a. *Is X_n a Markov Chain? Why or why not?*

b. *Another sequence Y_n is defined by $Y_n = X_n + X_{n-1}$. Is Y_n a Markov process? Why or why not?*

c. *Another sequence Z_n is defined by $Z_n = (X_n, X_{n-1})$. Is Z_n a Markov process? Why or why not?*

Problem 1.2 *Finite State Markov Chain*

Bob goes to Las Vegas. He does not want to lose a lot of money so decides to gamble with only \$3 and to stop playing if he loses these \$3 dollars or reaches \$5. He approaches to a roulette wheel, which contains 18 red, 18 black and 2 green holes. He decides to always bet \$1 on red on the roulette wheel.

a. *Formulate the Markov Chain corresponding to the amount of dollars that Bob has and classify the states.*

b. *Find the transition probability matrix of the Markov Chain. Determine its eigenvalues and eigenvectors. Express the row vector containing probability that Bob has \$i at the end of n-th game for $i = 0, 1, 2, 3, 4, 5$ in terms of the eigenvalues and eigenvectors.*

   (Use MATLAB to help yourself out)

c. *Find the steady state probability that Bob has \$i at the end of n-th game for $i = 0, 1, 2, 3, 4, 5$.*

d. *Repeat part (b) and (c) for the case when Bob starts with \$2 and stops playing if he loses all his money or has \$5.*

Problem 1.3 *Processing Jobs*

Consider a single job processor with a queue. Suppose that time is slotted into durations τ and jobs arrive at the beginning of the time slot as a Bernoulli process B_k with an independent probability $\lambda\tau$ that a given time slot will contain a job. The jobs will accumulate in a first-come first serve queue and the currently active job had an independent probability $q = \rho\tau$ of being completed in this time slot. If a job is completed at time $k\tau$, then it is removed from the queue and sent on to the next processor and we can say that $Y_k = 1$. If no job is completed then $Y_k = 0$.

a. *Give an explicit model for the above.*

b. What are the steady state probabilities for the queue size? What is the condition required for steady state to even exist?

c. Suppose that ρ and λ are such that the queue is stable (ie. has a steady state distribution). Furthermore, suppose that the queue is in steady state. What is the expected time till the next job is completed?

d. Let τ go to zero. What does the process of jobs coming into the processor look like?

e. Let τ go to zero. What is the steady state distribution of the queue sizes?

f. Let τ go to zero and assume that the queue is in steady state. What is the expected time till the next job is completed?

g. Bonus: Let τ go to zero and assume that the queue is in steady state. What is the pdf for the time of the next job completion?