EECS 126: Probability and Random Processes
Problem Set 6
Due on Thursday, March 3rd 2005 in class

Problem 14. Alexei is vacationing in Monte Carlo. The amount X (in dollars) he takes to the casino each evening is a random variable with a PDF of the form

$$f_X(x) = \begin{cases} ax, & \text{if } 0 \le x \le 40, \\ 0, & \text{otherwise.} \end{cases}$$

At the end of each night, the amount Y that he has when leaving the casino is uniformly distributed between zero and twice the amount that he came with.

(a) Determine the joint PDF $f_{X,Y}(x, y)$.

(b) What is the probability that on a given night Alexei makes a positive profit at the casino?

(c) Find the PDF of Alexei's profit $Y - X$ on a particular night, and also determine its expected value.

Problem 20. Your driving time to work is between 30 and 45 minutes if the day is sunny, and between 40 and 60 minutes if the day is rainy, with all times being equally likely in each case. Assume that a day is sunny with probability 2/3 and rainy with probability 1/3.

(a) Find the PDF, the mean, and the variance of your driving time.

(b) On a given day your driving time was 45 minutes. What is the probability that this particular day was rainy?

(c) Your distance to work is 20 miles. What is the PDF, the mean, and the variance of your average speed (driving distance over driving time)?

Problem 6. The transform and the mean associated with a discrete random variable X are given by

$$M(s) = ae^s + be^{4(e^s - 1)}, \qquad \mathbf{E}[X] = 3.$$

Find:

(a) The scalar parameters a and b.

(b) $p_X(1)$, $\mathbf{E}[X^2]$, and $\mathbf{E}[2^X]$.

(e) $\mathbf{P}(X+Y=2)$, where Y is a random variable that is independent of X and is identically distributed with X.

Problem 17. Let Y be exponentially distributed with parameter 1, and let Z be uniformly distributed over the interval $[0,1]$. Use convolution to find the PDF of $|Y-Z|$.

Problem 13. **The z-Transform.** The z-transform associated with a discrete random variable X is a function $Q_X(z)$ of a free parameter z, defined by

$$Q_X(z) = \mathbf{E}[z^X].$$

(a) Show that the transforms $M_X(s)$ and $Q_X(z)$ are related by

$$M_X(s) = Q_X(e^s).$$

(b) Show the moment-generating properties

$$Q_X(1) = 1, \qquad \frac{d}{dz}Q_X(z)\bigg|_{z=1} = \mathbf{E}[X], \qquad \frac{d^2}{dz^2}Q_X(z)\bigg|_{z=1} = \mathbf{E}[X^2] - \mathbf{E}[X].$$

Problem 19. Consider two independent and identically distributed discrete random variables X and Y. Assume that their common PMF, denoted by $p(x)$, is symmetric around zero, i.e., $p(x) = p(-x)$ for all x. Show that the PMF of $X+Y$ is also symmetric around zero and is largest at zero. *Hint:* Use the Schwarz inequality: $\sum_k (a_k b_k) \leq \left(\sum_k a_k^2\right)^{1/2}\left(\sum_k b_k^2\right)^{1/2}$.