UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: PROBABLITY AND RANDOM PROCESSES

Problem Set 4
Spring 2006

Issued: Friday, February 10, 2006 **Due:** Friday, February 17, 2006

Reading: Bertsekas & Tsitsiklis, §2.2—§2.6.

Problem 4.1
Two fair three-sided dice are rolled simultaneously.

(a) Let X be the sum of the two rolls. Calculate the PMF, the expected value, and the variance of X.

(b) Someone proposes to give you in dollars five times the amount of the sum X that you roll, if you pay a dollars in advance. What should be the amount a in order for you to expect to break even?

(c) Repeat parts (a) and (b) for the case where X is the square of the sum of the two rolls.

Problem 4.2
Random variables X and Y have the joint PMF

$$p_{X,Y}(x, y) = \begin{cases} cxy & , \quad x \in \{1, 2, 4\} \quad \text{and} \quad y \in \{1, 3\} \\ 0 & , \quad \text{otherwise.} \end{cases}$$

(a) What is the value of the constant c?

(b) What is $\mathbb{P}(Y < X)$?

(c) What is $\mathbb{P}(Y > X)$?

(d) What is $\mathbb{P}(Y = X)$?

(e) What is $\mathbb{P}(Y = 3)$?

(f) Find the marginal PMFs $p_X(x)$ and $p_Y(y)$.

(g) Find the expectations $\mathbb{E}[X]$ and $\mathbb{E}[Y]$.

(h) Find the variances $\text{var}(X)$ and $\text{var}(Y)$.

Problem 4.3
(Optional: not to be graded) Prove the following basic properties of the expectation.

(a) If $X \geq 0$ then $\mathbb{E}[X] \geq 0$.

(b) If $X \geq Y$ then $\mathbb{E}[X] \geq \mathbb{E}[Y]$.

(c) $|\mathbb{E}[X]| \leq \mathbb{E}[|X|]$.

Problem 4.4

Let random variable X have a given PMF $p_X(x)$. We wish to guess, or *estimate*, the value of X before performing the actual experiment. Denoting our guess by $\hat{x}$, define the *mean square estimation error* by

$$e(\hat{x}) = \mathbb{E}\left[(X - \hat{x})^2\right].$$

Show that $e(\hat{x})$ is minimized by $\hat{x} = \mathbb{E}[X]$.

Problem 4.5

Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables N and K are defined by

$N =$ down-face value on the throw of 4-sided die

$K =$ total number of heads resulting from the coin flips.

Determine and sketch each of the following functions for all values of their arguments:

(a) $p_N(n)$ (b) $p_{K|N}(k \mid 2)$ (c) $p_{N|K}(n \mid 2)$ (d) $p_K(k)$.

Also determine

(e) the conditional PMF for random variable N, given that the experimental value of K is an odd number.

Problem 4.6

A receiver receives a sequence of signals sent by a particular transmitter. For each signal (independently of the others), there is a $\frac{1}{6}$ probability that the signal received will be a 1, a $\frac{1}{6}$ probability that it will be a 0, and a $\frac{2}{3}$ probability that the signal will be unintelligible. Define the random variable X to be the number of signals (intelligible or not) received up to and including the first 1. Define Y to be the number of signals received up to an including the first 0.

(a) Find $\mathbb{E}[X]$.

(b) Find $\mathbb{E}[X|Y = 1]$.

(c) Find $E[X|Y = 5]$.