Problem 1. (Fall 2008, MT2) Given $X \in \{0, 1\}$, the random variable $Y$ is exponentially distributed with rate $3X + 1$.

(a) Assume $P(X = 1) = p$ and $P(X = 0) = 1 - p$. Find the MAP estimate of $X$ given $Y$.

(b) Find the MLE of $X$ given $Y$.

(c) Solve the hypothesis testing problem of $X$ given $Y$ with a probability of false alarm at most 0.1. That is, find $\hat{X}$ as a function of $Y$ that maximizes $P[\hat{X} = 1|X = 1]$ subject to $P[\hat{X} = 1|X = 0] \leq 0.1$.

(d) For what value of $p$ does one have the same solution for (a) and (c)?

Problem 2. Let $Y \in \{A, B, C\}$ such that $P(Y = A|X = 1) = 0.2$, $P(Y = B|X = 1) = 0.2$, $P(Y = C|X = 1) = 0.6$ and $P(Y = A|X = 0) = 0.2$, $P(Y = B|X = 0) = 0.5$, $P(Y = C|X = 0) = 0.3$. We would like to solve a hypothesis testing with probability of false alarm at most $\beta$.

(a) Find the likelihood ratio.

(b) Find PCD and PFA for $\lambda = 1, 1.4, 2, 2.1$.

(c) Draw the ROC.