UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

**Discussion 8**
Spring 2018

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1. **Poisson Branching**

   Consider a branching process such that at generation $n$, each individual in the population survives until generation $n+1$ with probability $0 < p < 1$, independently, and a Poisson number (with parameter $\lambda$) of immigrants enters the population. Let $X_n$ denote the number of people in the population at generation $n$.

   (a) Suppose that at generation 0, the number of people in the population is a Poisson random variable with parameter $\lambda_0$. What is the distribution at generation 1? What is the distribution at generation $n$?

   (b) What is the distribution of $X_n$ as $n \to \infty$? What if at generation 0, the number of individuals is an arbitrary probability distribution over the non-negative integers; does the distribution still converge? (Justify the convergence rigorously.)

**2. Poisson Practice**

Let $(N(t), t \geq 0)$ be a Poisson process with rate $\lambda$. Let $T_k$ denote the time of $k$-th arrival, for $k \in \mathbb{N}$, and given $0 \leq s < t$, we write $N(s,t) = N(t) - N(s)$. Compute:

(a) $\mathbb{P}(N(1) + N(2,4) + N(3,5) = 0)$.

(b) $\mathbb{E}(N(1,3) \mid N(1,2) = 3)$.

(c) $\mathbb{E}(T_2 \mid N(2) = 1)$.

**3. System Shocks**

For a positive integer $n$, let $X_1, \ldots, X_n$ be independent exponentially distributed random variables, each with mean 1. Let $\gamma > 0$.

A system experiences shocks at times $k = 1, \ldots, n$. The size of the shock at time $k$ is $X_k$.

(a) Suppose that the system fails if any shock exceeds the value $\gamma$. What is the probability of system failure?

(b) Suppose instead that the effect of the shocks is cumulative, i.e., the system fails when the total amount of shock received exceeds $\gamma$. What is the probability of system failure?