Problem 1. Expected Squared Arrival Times
Let \((N(t), t \geq 0)\) be a Poisson process with arrival instants \((T_n, n \in \mathbb{N})\), where \(0 < T_1 < T_2 < \cdots\). Find \(E(\sum_{k=1}^{3} T_k^2 \mid N(1) = 3)\).

Problem 2. Minimum and Maximum of Exponentials
Let \(\lambda_1, \lambda_2 > 0\), and \(X_1 \sim \text{Exponential}(\lambda_1), X_2 \sim \text{Exponential}(\lambda_2)\) are independent. Also, define \(U := \min(X_1, X_2)\) and \(V := \max(X_1, X_2)\). Show that \(U\) and \(V - U\) are independent.
Problem 3. Spatial Poisson Process

A two-dimensional Poisson process of rate $\lambda > 0$ is a process of randomly occurring special points in the plane such that (i) for any region of area $A$ the number of special points in that region has a Poisson distribution with mean $\lambda A$, and (ii) the number of special points in non-overlapping regions is independent. For such a process consider an arbitrary location in the plane and let $X$ denote its distance from its nearest special point (where distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is defined as $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$). Show that:

(a) $\mathbb{P}(X > t) = \exp(-\lambda \pi t^2)$ for $t > 0$.

(b) $\mathbb{E}[X] = \frac{1}{2\sqrt{\lambda}}$. 