Problem 1. Answer the following questions:

1. Show that the probability that exactly one of the events $A$ and $B$ occurs is
   \[ P(A) + P(B) - 2P(A \cap B). \]

2. If $A$ is independent of itself, show that $P(A) = 0$ or $1$.

Problem 2. You know that, at least one of the events $A_r$ (for $r \in \{1, \ldots, n\}$, where $n$ is an integer $\geq 2$) is certain to occur but certainly no more than two occur. Show that if the probability of occurrence of any single event is $p$, and the probability of joint occurrence of any two distinct events is $q$, we have $p \geq 1/n$ and $q \leq 2/[n(n-1)]$.

Problem 3. Consider a sphere that has $\frac{1}{10}$ of its surface colored blue, and the rest is colored red. Show that, no matter how the colors are distributed, it is possible to inscribe a cube in the sphere with all of its vertices red.

Hint: Carefully define some relevant events.
Problem 4. [Extra] The Countable Union Bound

Let $A_1, A_2, \ldots$ be a countable sequence of events. Prove that the union bound holds for countably many events:

$$\Pr\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} \Pr(A_i).$$