UC Berkeley EE128 Homework #9

Due on 12/04/06 at 11am

(1) Textbook problem 7.44

44. The linearized equations of motion of the simple pendulum in Fig. 7.95 are

$$\ddot{\theta} + \omega^2\theta = u.$$

a) Write the equations of motion in state-space form.
b) Design an estimator (observer) that reconstructs the state of the pendulum given measurements of $\dot{\theta}$. Assume $\omega = 5$ rad/sec, and pick the estimator roots to be at $s = -10 \pm 10j$.
c) Write the transfer function of the estimator between the measured value of $\dot{\theta}$ and the estimated value of θ.
d) Design a controller (that is, determine the state feedback gain $\mathbf{K}$) so that the roots of the closed-loop characteristic equation are at $s = -4 \pm 4j$.

(2) Textbook problem 7.46

46. A certain process has the transfer function $G(s) = 4/(s^2 - 4)$.
a) Find $\mathbf{F}$, $\mathbf{G}$, and $\mathbf{H}$ for this system in observer canonical form.
b) If $u = -\mathbf{Kx}$, compute $\mathbf{K}$ so that the closed-loop control poles are located at $s = -2 \pm 2j$.
c) Compute $\mathbf{L}$ so that the estimator-error poles are located at $s = -10 \pm 10j$.
d) Give the transfer function of the resulting controller (for example, using Eq. (7.177)).
e) What are the gain and phase margins of the controller and the given open-loop system?

(3) Textbook problem 7.50

50. Unstable equations of motion of the form,

$$\ddot{x} = x + u,$$

arise in situations where the motion of an upside-down pendulum (such as a rocket) must be controlled.
a) Let $u = -Kx$ (position feedback alone), and sketch the root locus with respect to the scalar gain K.
b) Consider a lead compensator of the form,

$$U(s) = K\frac{s+a}{s+10}X(s).$$

Select a and K so that the system will display a rise time of about 2 sec and no more than 25% overshoot. Sketch the root locus with respect to K.
c) Sketch the Bode plot (both magnitude and phase) of the uncompensated plant.
d) Sketch the Bode plot of the compensated design, and estimate the phase margin.
e) Design state feedback so that the closed-loop poles are at the same locations as those of the design in part (b).
f) Design an estimator for x and $\dot{x}$ using the measurement of $x = y$, and select the observer gain $\mathbf{L}$ so that the equation for $\tilde{x}$ has characteristic roots with a damping ratio $\zeta = 0.5$ and a natural frequency $\omega_n = 8$ rad/sec.
g) Draw a block diagram of your combined estimator and control law, and indicate where $\hat{x}$ and $\dot{x}$ appear. Draw a Bode plot for the closed-loop system, and compare the resulting bandwidth and stability margins with those obtained using the design of part (b).