LAB 5: SIMPLE SYSTEM IDENTIFICATION

ANIL ASWANI

1. Overview

The task of system identification is one that is commonly encountered by the control engineer. Often times, a physical system is too complicated to be modeled (or modeled reliably). In these cases, it is often acceptable to heuristically determine a model using physical insights and measurements. This is what we do with the MagLev system in Lab 5.

In general, there are many considerations to make when doing system identification. We will only concern ourselves with some of the simpler issues. There are two approaches to use that will give us similar results in this lab. One approach is to take measurements of the physical system, fit the data to a model based on physical intution, and then linearize the model. This is the preferred approach. The other approach is to take measurments, make crude estimates of the slopes of the function, and then use the estimates to develop a linear model.

2. Curve Fitting

After we have made measurements of our system, we still have to do something useful with the data. We can fit the data to a curve, and then linearize the curve. The natural question to ask is: what curve should we fit our data to? One technique is to fit our data to a general curve such as a polynomial of fixed degree or a series of sinusoids. Fortunately, in our lab we can do better and use our physical intution.

The system we are concerned with is a MagLev. Looking at the Biot-Savart law, we might expect the force due to the electro-magnet to be proportional to the current and inversely propotional to the distance (this is sometimes refered to by saying that the Biot-Savart law is an inverse-square law). Thus, we expect our force to be of the form:

$$f(i,x) = \frac{ki}{r^2}$$

The question is now: how do we determine the coefficient k? The simplest way to do this is with a linear least-squares fit. Linear least-squares is one way of getting a "best-fit" of a curve to data. The general formula for a least-squares fit is $c = (A^T A)^{-1} A^T y$, where y is our output measurmeants, c is the coefficients we need to determine, and A is a matrix of our data points substituted into our curve.

We will make this more concrete with an example from the lab:

2.1. Example: Linear Least-Squares for MagLev.

Suppose that the hypothetical measurements that we have made on our system are given in Table 1. Note that r is the absolute distance between the electro-magnet and the ball (i.e., $r = x + 6\ mm$).

Since we suspect that our data matches the model $f(i,x) = \frac{ki}{r^2}$, we will substitute our values of i and r into $f(i,x)$. Taking the first set of data points, we have $f(0.5, 0.003) = \frac{(0.5)k}{0.003^2} = (55600)k = 0.26$. Doing this for all of our data points, we get:

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$$\begin{pmatrix} 55600 \\ 20000 \\ 10200 \\ 111000 \\ 40000 \\ 20400 \\ 167000 \\ 60000 \\ 30600 \end{pmatrix} k = \begin{pmatrix} 0.26 \\ 0.11 \\ 0.09 \\ 0.60 \\ 0.23 \\ 0.13 \\ 0.79 \\ 0.30 \\ 0.10 \end{pmatrix}$$

We can rewrite this as $Ak = y$. Remember that the linear least-squares estimate is $c = (A^T A)^{-1} A^T y$. Doing this calculation in MATLAB gives $c = 5e - 6$.

Now that we have fit our data to a curve, we should plot our model against our data to compare how well our model fits. If we are satisfied with our fit, we can proceed with the linearization and the other calculations.

3. Slope Estimation

We can use a cruder method to estimate the slope of our function $f(i, x)$ for the linearization. By using the points aboutour point of interest, such as $x = 0\ mm$ (or equivalently $r = 6\ mm$), we can estimate the slope about $x = 0\ mm$. This method is not as good as the earlier method, because we are essentially throwing away our other measured values by not using them. Many consider this method to be invalid for linearization and curve fititng. For completeness, an example using this method is presented below:

3.1. **Example: Slope Estimation for MagLev.** Using the data given in Table 1, we will linearize the system about $x = 0\ mm$ (or equivalently $r = 6\ mm$). The estimated slope in the i-direction is:

$$m_i \approx \frac{\Delta f}{\Delta i} = \frac{0.60 - 0.26}{1.0 - 0.5} = 0.68$$

and the estimated slope in the x-direction is:

$$m_x \approx \frac{\Delta f}{\Delta x} = \frac{0.11 - 0.09}{0.005 - 0.007} = -10$$

From this, we can write the linearization. Note that we can improve this technique by averaging more values in, but eventually this becomes similar to doing linear least-squares.

i (A)	r (m)	F (N)
0.5	0.003	0.26
0.5	0.005	0.11
0.5	0.007	0.09
1.0	0.003	0.60
1.0	0.005	0.23
1.0	0.007	0.13
1.5	0.003	0.79
1.5	0.005	0.30
1.5	0.007	0.10

Table 1. Hypothetical Data for the MagLev System

4. Conclusion

System identification is a highly-non-linear process. There are multiple stages of making measurements, doing curve-fitting, refining the developed model, and doing verification (making new measurements and seeing how close the model fits the new data) of the model. A good starting point for more information is [1], though much of the material is beyond the scope of our class.

5. References

[1] Lennart Ljung, "Identification of Linear and Nonlinear Dynamical Systems Short Course, Fall 2005," (http://jagger.me.berkeley.edu/sysid/)