# Rules for Plotting a Root Locus

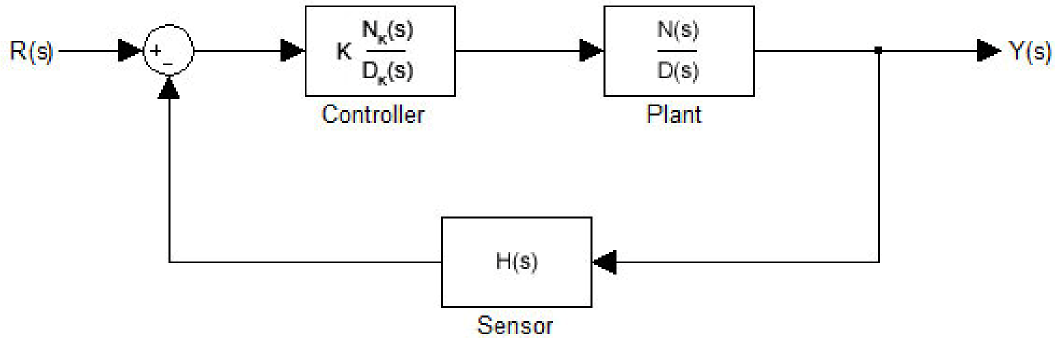


**Figure 1:** General Feedback System with Controller

$$\frac{Y(s)}{R(s)} = \frac{KN_K(s)N(s)}{D_K(s)D(s)+KH(s)N_K(s)N(s)}$$

So characteristic equation $\Delta(s) = D_K(s)D(s) + KH(s)N_K(s)N(s) = 0$.

Which is of the form $\boxed{a(s) + Kb(s) = 0}$, where $a(s) = D_K(s)D(s)$ and $b(s) = H(s)N_K(s)N(s)$.

Or, $1 + K\frac{H(s)N_K(s)N(s)}{D_K(s)D(s)} = 0$.

Let $L(s) = \frac{H(s)N_K(s)N(s)}{D_K(s)D(s)}$, and we are left with $\boxed{1 + KL(s) = 0}$

Assume the system has $m$ zeros and $n$ poles.

**Rule 1:** (Poles and Zeros)

- $n$ branches of the locus start at the Poles of $L(s)$
  + Start at $K = 0$ and you get $\Delta(s) = a(s) = D_K(s)D(s) = 0$, which are the poles.
- $m$ of these branches end on the zeros of $L(s)$
  + End as $K \rightarrow \infty$, so effect of $a(s)$ becomes negligible compared to $b(s)$ and you get $\Delta(s) \approx Kb(s) = 0$, so solving for $b(s) = H(s)N_K(s)N(s) = 0$ yields the zeros.

**Rule 2:** (Real axis)

- The loci are on the **real axis** to the left of an odd number of poles and zeros
  + Take a test point on the real axis. To be in the locus, the phase of $L(s)$ has to be 180°.
  + Complex poles cancel with each other because 1) they come in vertical pairs and 2) have offsetting phases.

**Rule 3:** (Asymptotes)

- For large s and K, $n$ - $m$ of the loci are asymptotic to lines at angles $\Phi_l$ radiating out from the center points $s = \alpha$ on the real axis.
  - + $n$ - $m$ = # of poles that DON'T end up approaching a zero = # of different asymptotes
  - + All asymptotes intersect the real axis at same point $(\alpha, 0)$.

$$\alpha = \frac{\sum p_i - \sum z_i}{n-m} \qquad \phi_l = \frac{180^o + 360^o(l-1)}{n-m}, \text{ for } l = 1,2,\ldots,n\text{-}m$$

**Rule 4:** (Angles of arrival and departure)

- The angle(s) of departure of a branch of the locus from a pole of multiplicity $q$ is given by:

$$q\phi_{l,dep} = \sum \psi_i - \sum \phi_i - 180^o - 360^o(l-1)$$

- The angle(s) of arrival of a branch at a zero of multiplicity $q$ is given by:

$$q\psi_{l,arr} = \sum \phi_i - \sum \psi_i + 180^o + 360^o(l-1)$$

**Rule 5:** ($j\omega$-axis crossings)

- The locus crosses the $j\omega$ axis at points where the Routh criterion shows a transition from roots in the LHP to roots in the RHP.

**Rule 6:** (Break-in and break-out points)

- The locus will have multiple roots at points where the derivative is zero, or

$$\left(b\frac{da}{ds} - a\frac{db}{ds}\right) = 0$$

- Real-valued zeros of this expression are the break-in and break-out points along the real axis.