Discussion 4: Root Locus Example

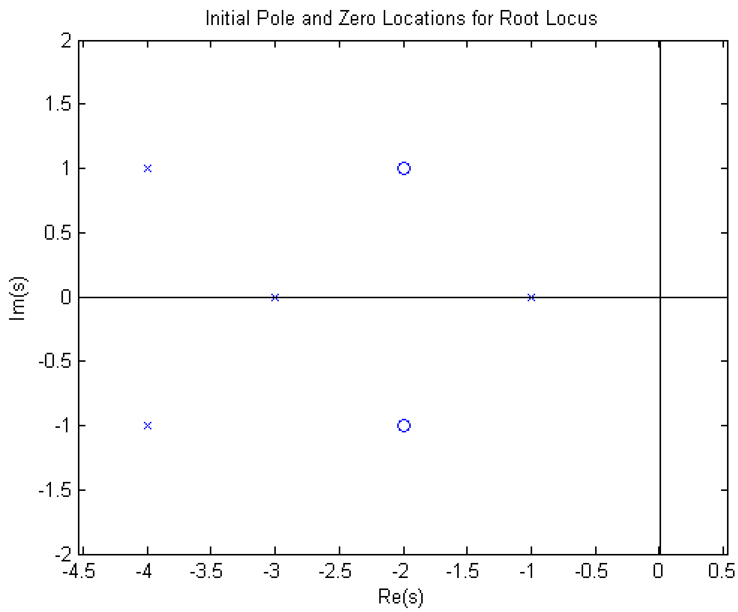

Figure 1: Initial pole and zero locations

<u>Rule 1</u>: Given poles @ s = -1, -3, -4+j, -4-j and zeros @ s = -2+j, -2-j

So $L(s) = \frac{((s+2)^2+1^2)}{(s+1)(s+3)((s+4)^2+1^2)} = \frac{s^2+4s+5}{(s^2+4s+3)(s^2+8s+17)} = \frac{s^2+4s+5}{s^4+12s^3+52s^2+92s+51}$

<u>Rule 2</u>: Real axis on root locus only between poles at $s = -1$ and $s = -3$.

<u>Rule 3</u>: $n - m = 4 - 2 = 2$, so $\phi = \pm 90^o$

$\alpha = \frac{-4-4-3-1-(-2-2)}{2} = \frac{-8}{2} = -4$

<u>Rule 4</u>: Angle of departure for top pole ($s = -4 + j$):

$$1\phi_{1,dep} = (180 + 135) - (90 + 135 + \tan^{-1}(-1/4)) - 180$$

$\phi_{1,dep} = -90 - 165.96 = -255.96^o = 104.04^o$, so just left of vertical

Angle of departure for bottom pole ($s = -4 - j$):

$$\phi_{2,dep} = -104.04^o$$

Angle of arrival for top zero ($s = -2 + j$):

$$1\psi_{1,arr} = (0 + 45 + 45 + 135) - (90) + 180 = 135 + 180 = 315 = -45^o$$

Angle of arrival for bottom zero ($s = -2 - j$):

$$\psi_{2,arr} = 45^o$$

<u>Rule 5</u>: With the two asymptotes heading vertically at $s = -4$ and the other poles cancelling with the zeros, we do not expect to see any $j\omega$-axis crossings.

Rule 6: Used the MATLAB code below to get $\left(b\frac{da}{ds} - a\frac{db}{ds}\right) = 0$

```
%%% disc4.m
%%% Justin Hsia - Fall 2008
%%% Calculating break-in/break-out points for Discussion 4 root locus
%%% example.

b = poly([-2+j -2-j]);
a = poly([-1 -3 -4+j -4-j]);

rule6 = conv(b,polyder(a))-conv(a,polyder(b));
r = roots(rule6)

clf
rlocus(tf(b,a));
axis equal
```

The MATLAB output is:

```
>> disc4
r =
 -3.7004 + 0.3057i
 -3.7004 - 0.3057i
 -1.3940 + 1.7841i
 -1.3940 - 1.7841i
 -1.8112
```

The only real zero is at $s = -1.8112$. This is the break-out point for the two poles on the real axis.

Completed root locus:

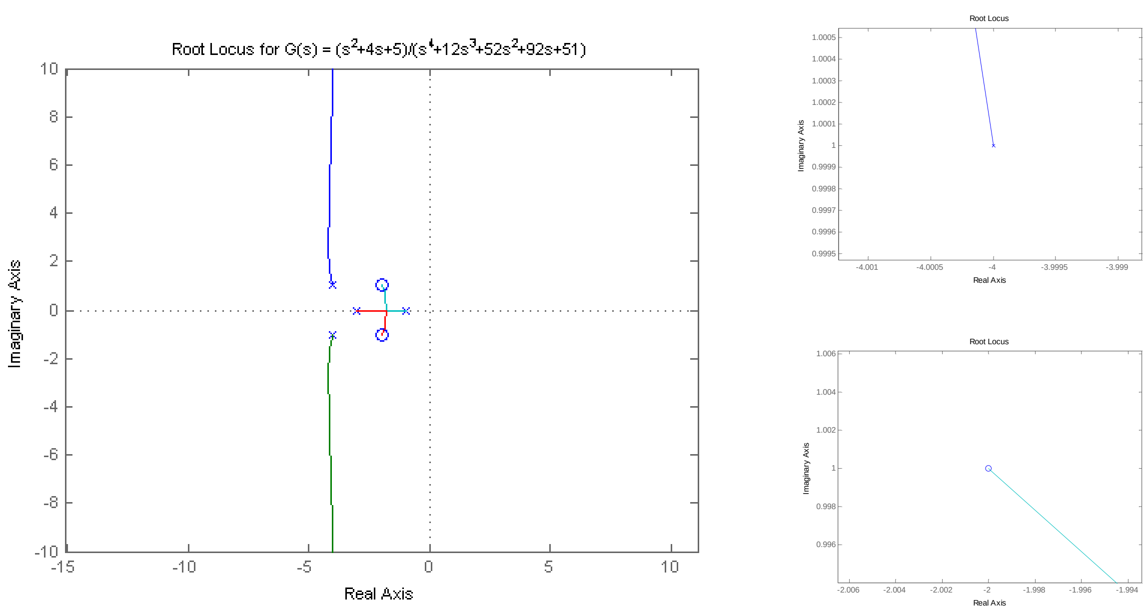

Figure 2: (left) Complete root locus; (top-right) zoomed-in view of angle of departure for top pole; (bottom-right) zoomed-in view of angle of arrival for top zero.