EECS 128 Introduction to Control Design Techniques
Final Exam Practice Problems

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Friday December 19, 12.30-3.30pm

- You have 3 hours to complete the final exam; you may use 1 8.5×11 crib sheet (double sided) during the exam.
- The problems below are for practice: the actual final exam problems will be such that you will not need a calculator or MATLAB.
- The exam questions will focus on material from after the midterm, and the questions below are not representative of the scope of our coverage since the midterm.

Problem 1: Controllability and observability of interconnections.

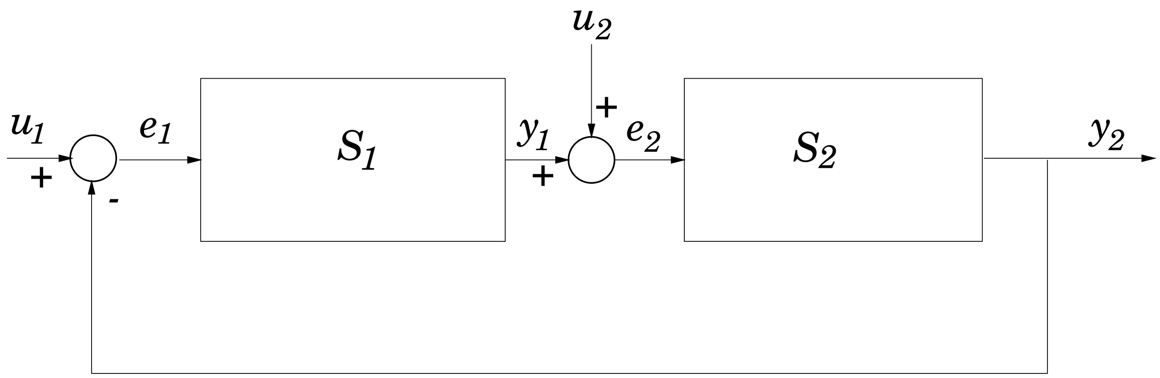

Figure 1: Two interconnected systems.

Consider the two systems shown in Figure 1, in which S_1 represents the system:

$$\dot{x}_1 = A_1 x_1 + B_1 e_1 \quad (1)$$
$$y_1 = C_1 x_1 \quad (2)$$

where $A_1 \in \Re^{n_1 \times n_1}$, $B_1 \in \Re^{ni_1}$, and $C_1 \in \Re^{no_1}$; and S_2 represents the system:

$$\dot{x}_2 = A_2 x_2 + B_2 e_2 \quad (3)$$
$$y_2 = C_2 x_2 \quad (4)$$

where $A_2 \in \Re^{n_2 \times n_2}$, $B_2 \in \Re^{ni_2}$, and $C_2 \in \Re^{no_2}$. Assume that S_1 and S_2 are both controllable and observable.

(a) Write a state space representation of the system shown in Figure 1, with input $[u_1\ u_2]^T$, output $[y_1\ y_2]^T$, and state vector $[x_1\ x_2]^T$.

(b) Prove that the composite system is both controllable and observable.

Problem 2: Design.

Consider the following system, which represents the heading dynamics of a tricycle landing gear on the ground, with heading actuator on the front wheel as input:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$
$$y = [1\ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(a) What is the steady state error in the output to a *step input* $u(t)$?

(b) Design an *output* feedback controller, $u(t) = -ky(t) + r(t)$, so that a step input at the reference $r(t)$ results in zero steady state error.

(c) Design a state feedback controller $u(t) = -[F_1\ F_2][x_1\ x_2]^T + \overline{N}r(t)$ (assuming the state may be measured) so that the eigenvalues of the closed loop system dynamics are at: $(-2, -2)$.

(d) Now, assuming that only the output may be measured, design a full state observer so that the error between estimated and actual state is bounded above by e^{-4t}.

(e) Now consider the complete closed loop design of (c), (d) above, with controller and observer in the feedback loop. What is the steady state error to a *step input* $r(t)$?

Problem 3: Observer design (10 points).

You are given a system

$$\begin{aligned}\dot{X} &= AX + BU \\ Y &= CX\end{aligned}$$

which is both controllable and observable. You design a full order observer with gain matrix T such that $(A - TC)$ is stable; denoting observer state $\hat{Z}$, you design a feedback gain matrix F, such that $U = R - F\hat{Z}$, where R is the reference input and $(A - BF)$ is stable. Denote the state estimate error as $E = \hat{Z} - X$.

(a) Derive the transfer function from the reference input R to the output Y, in terms of the plant and gain matrices. What is striking about this transfer function?

(b) Derive the transfer function from the reference input R to the error E. Comment about this transfer function.

(c) Suppose that the plant changes by a small amount δA. What is the resulting $H_{EU}(s)$? Does the error E still go to zero as $t \to \infty$?