EECS 128 Introduction to Control Design Techniques
Problem Set 1

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Problem 1: Lotka-Volterra Predator-Prey Equations. Count Vito Volterra was an Italian mathematician (1860-1940), who developed a mathematical model to explain the results of a statistical study of fish populations in the Adriatic Sea. In particular, his model explains the increase in predator fish (and corresponding decrease in prey fish) which he observed during the World War I period. Volterra produced a series of models for the interaction of two or more species. Alfred J. Lotka was an American biologist and actuary who independently produced many of the same models.

One of the simplest of their models takes the form

$$\dot{x}_1 = ax_1 - bx_1x_2 \tag{1}$$
$$\dot{x}_2 = -dx_2 + cx_1x_2 \tag{2}$$

where $x_1 \geq 0$ denotes the sardine (prey) population and $x_2 \geq 0$ denotes the shark (predator) population. a, b, c, and d are all positive constants. Note that the equations model the facts that: sardines multiply faster as they increase in number; the number of sardines decreases as both the sardine and shark population increases; sharks increase in number at a rate proportional to the number of shark-sardine encounters.

(a) Determine all equilibria of this system.

(b) Linearize the system about each equilibrium that you found in part (a), and write the results in the form of a first order vector differential equation $\dot{\delta x} = A\delta x$.

(c) Program your model into MATLAB, choosing representative values of a, b, c, and d. Show, using simulation, how the shark and sardine populations evolve for the following three initial conditions:

- no sardines, a few sharks
- a few sardines, no sharks
- $\frac{d}{c}$ sardines, $\frac{a}{b}$ sharks

Problem 2: A magnetically suspended steel ball, linearized.

The simplified dynamics of a magnetically suspended steel ball are given by:

$$m\ddot{y} = mg - c\frac{u^2}{y^2} \tag{3}$$

where the input u represents the current supplied to the electromagnet, y is the vertical position of the ball, which may be measured by a position sensor, g is gravitational acceleration, m is the mass of the ball, and c is a positive constant such that the force on the ball due to the electromagnet is $c\frac{u^2}{y^2}$. Assume a normalization such that $m = g = c = 1$.

(a) Using the states $x_1 = y$ and $x_2 = \dot{y}$ write down a nonlinear state space description of this system.

(b) What *equilibrium control input* u_e must be applied to suspend the ball at $y = 1$ m?

(c) Write the linearized state space equations for state and input variables representing perturbations away from the equilibrium of part (b).

Problem 3: Linearity.

(a) For each of the following systems H, indicate whether or not the system is (i) Linear, (ii) Time-Invariant. Justify your answers.

1) $y(t) = H(u(t)) = e^{u(t)}$
2) $y(t) = H(u(t)) = au(t) + b$
3) $y(t) = H(u(t)) = au_1(t) + bu_2(t)$, with $u(t) = [u_1(t)\ u_2(t)]^T$
4) $y(t) = H(u(t)) = u(-t)$
5) $y(t) = H(u(t)) = \int_0^t e^{-\sigma} u(t-\sigma) d\sigma$

(b) Consider an amplifier having input voltage $v_i(t)$ and output voltage $v_o(t)$ related by $v_o(t) = 8(v_i(t))^2$. Show that the amplifier is not linear. Derive a linearized model around operating point (v_{iQ}, v_{oQ}).

Problem 4: Transfer function models. Derive the transfer functions $\frac{V_o(s)}{V_i(s)}$ for each of the RC networks (i) and (ii) in Figure 1. Assume that $R_1 > 0$, $R_2 > 0$ denote resistor values, and $C > 0$ denotes capacitor values.

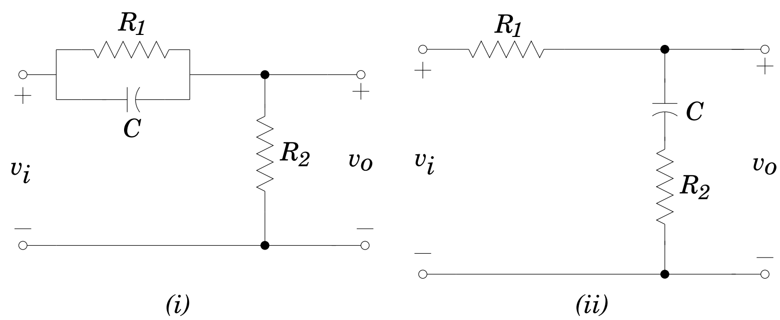

Figure 1: Two RC Networks

Problem 5. Given

$$Y(s) = \frac{a}{s^2 + a^2} \tag{4}$$

What is $\lim_{t \to \infty} y(t)$?