EECS 128 Introduction to Control Design Techniques
Problem Set 2

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Problem 1: Time response from transfer function.

A control system has the specifications: rise time $t_r \leq 0.01$ sec, overshoot $M_p \leq 16\%$, and steady state error to unit ramp $e_{ss} \leq 0.005$.

Sketch the allowable region in the s-plane for the dominant second order poles of an acceptable system.

If $Y/R = G/(1+G)$, what condition must $G(s)$ satisfy near $s = 0$ for the closed loop system to meet specifications – that is, what is the required low frequency behavior of $G(s)$?

Problem 2: Root Locus techniques.

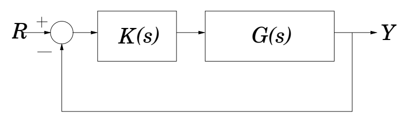

Figure 1: Unity Feedback System with proportional controller $K(s)$, plant $G(s)$

Consider the unity feedback system of Figure 1 with $K(s)$ given by $K(s) = 8\frac{s+\alpha}{s}$, and $G(s) = \frac{1}{(s+4)(s-1)}$. Use root locus methods to sketch the evolution of the closed loop poles for $\alpha \in [0, \infty]$.

Problem 3: Root Locus techniques.

Suppose you are given the plant

$$G(s) = \frac{1}{s^2 + (1+\alpha)s + (1+\alpha)}$$

where α is a system parameter that is subject to variations. Use root locus methods to determine what variations in α can be tolerated before instability occurs (note that α can be both positive and negative).

Problem 4.

Consider the RC networks (i) and (ii) from Problem 5 of Problem Set 1, and the transfer functions that you derived for each. You are given the plant $G(s) = \frac{1}{s^2}$ and you wish to design a compensator $K(s) = K \cdot K_o(s)$ as in Figure 2 below to improve the stability properties of the system. Sketch the root locus when $K_o(s) = 1$. Now, suppose you could choose for $K_o(s)$ either circuit (i) or circuit (ii) from part (a) to stabilize the system. Which would you select and why? Sketch the root locus of your resulting control system, for $R_1 = R_2 = 1$ and $C = 1$.

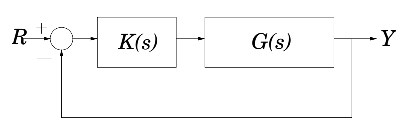

Figure 2: Unity Feedback System with proportional controller $K(s)$, plant $G(s)$

Problem 5: Speed control system for a magnetic tape drive.

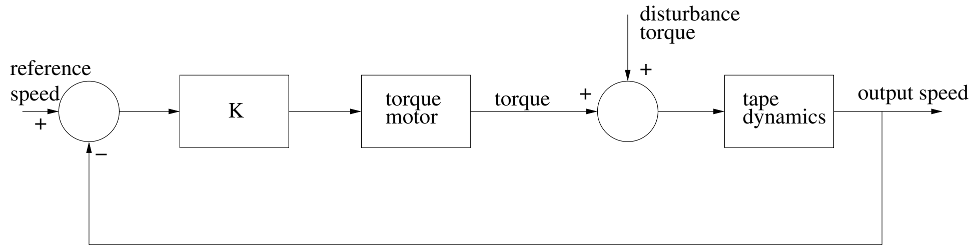

Figure 3: Speed control system for a magnetic tape drive

The speed control system for a magnetic tape drive is shown in Figure 3, with

$$\text{torque motor:}\frac{10}{0.5s+1} \quad \text{tape dynamics:}\frac{1}{Js+b}$$

where $J = 0.10$ kg $\cdot$ m^2 and $b = 1.00$ N$\cdot$ m$\cdot$ sec. Denote the reference speed as ω_r, the output speed as ω.

(a) With $\omega_r = 0$, what is the steady state error due to a step disturbance torque of 1 N$\cdot$ m? What must the amplifier gain K be in order to make the steady state error $e_{ss} \leq 0.01$ rad/sec?

(b) Using the gain K computed in (a), plot the poles of the closed loop system in the complex plane, and accurately sketch the time response $\omega(t)$ for a step input ω_r. Are you satisfied with these pole locations/this time response?

(c) Plot the region in the complex plane of acceptable closed loop poles corresponding to the specifications of a 1% settling time of $t_s \leq 0.1$ sec, and an overshoot $M_p \leq 5\%$.