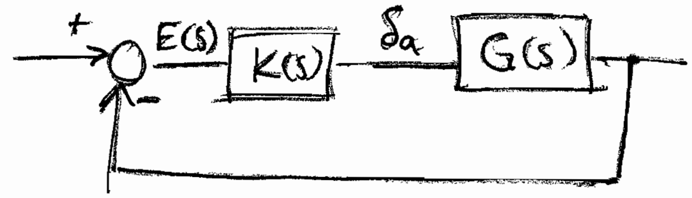
1. ψ_{ref}



$\psi(s)$

where $G(s) = \dfrac{\psi(s)}{\delta_a(s)} = \dfrac{V}{L} \cdot \dfrac{1}{s(\tau s+1)}$

$$\frac{\psi(s)}{\psi_{ref}(s)} = \frac{K(s)\,G(s)}{1 + K(s)\,G(s)}$$

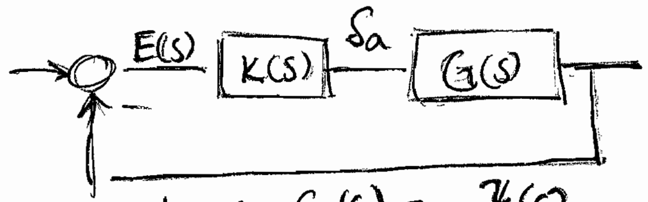
(a) $$e_{ss\_step} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\left(1 - \frac{\psi(s)}{\psi_{ref}(s)}\right)\psi_{ref}(s)$$

$$= \lim_{s\to 0} \cancel{s} \cdot \frac{1}{1+K(s)G(s)} \cdot \frac{1}{\cancel{s}}$$

$$\therefore\ e_{ss\_step} = \lim_{s\to 0} \frac{1}{1 + K(s)\frac{V}{L}\cdot\frac{1}{s(\tau s+1)}}$$

The simplest possible controller that achieves $e_{ss\_step} = 0$ is a pure gain $K(s) = K$.

(b) Now $\dot{\psi}_{ref}$



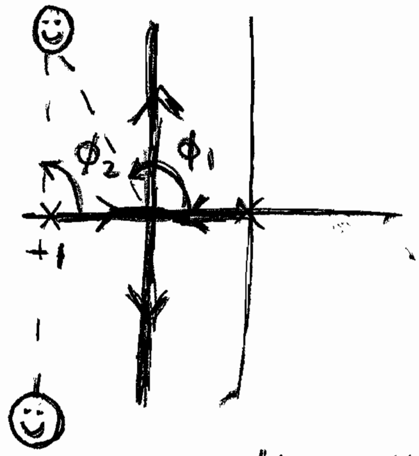
$\dot{\psi}(s)$

where $G(s) = \dfrac{\dot{\psi}(s)}{\delta_a(s)} = \dfrac{V}{L}\,\dfrac{1}{(\tau s+1)}$

$$e_{ss\_step} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{1+K(s)\frac{V}{L}\frac{1}{(\tau s+1)}} \cdot \frac{1}{\cancel{s}}$$

here, for $e_{ss\_step} = 0$, $K(s)$ needs a pole @ 0 $\Rightarrow$ $K(s) = \dfrac{1}{s}$ is a suitable controller

2. $G(s) = \dfrac{1}{s(s+1)}$



(a) Graphically, using only pure gain K, we see that the locus never goes through the point

$$☺ = -1 \pm \sqrt{3}\, j$$

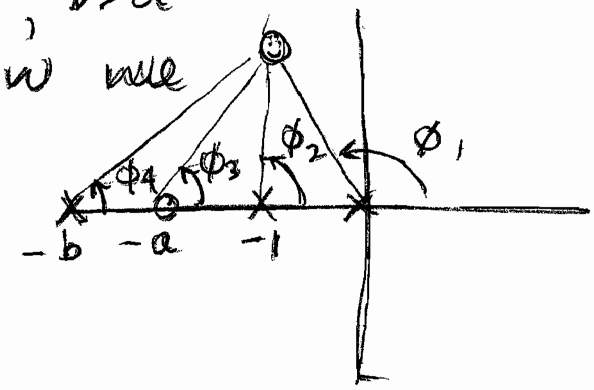
Mathematically, $\measuredangle G(s=-1\pm\sqrt{3}j) = -\phi_1 - \phi_2$

$$= -210° \neq -180°$$

∴ not on locus

(b) $\text{lead} = K_L \dfrac{s+a}{s+b}$, $b>a$

Let $\boxed{a=2}$ and now we must solve for b such that $s=-1\pm\sqrt{3}j$ is on the locus



$$\measuredangle G(s=☺) = -180° = -\phi_1 - \phi_2 + \phi_3 - \phi_4$$

$$= -210° + \tan^{-1}\left(\frac{\sqrt{3}}{2-1}\right) - \tan^{-1}\left(\frac{\sqrt{3}}{b-1}\right)$$

Solve .. $\boxed{b=4}$ ←

* note $a=2$ is arbitrary so this is one of many sol$^{\underline{n}}$s which will work

$$K_L = \left|\frac{1}{G_{OL}(s=-1\pm\sqrt{3}j)}\right| = \left|\frac{(-1+\sqrt{3}j)(\sqrt{3}j)(3+\sqrt{3}j)}{(1+\sqrt{3}j)}\right| = 6 \leftarrow$$

$$\Rightarrow \text{lead} = 6\,\frac{s+2}{s+4}.$$

3. (a) $G(s) = \dfrac{4}{s(s+0.5)}$ Require: $e_{ss\_ramp} < 0.02$

$\xi = 0.5$ for dom. CL poles

$w_n = 5$ rad/s "

$\Rightarrow$ 2nd order dominant poles:

$$s^2 + 2\xi w_n s + w_n^2 = 0$$

$$\Rightarrow s = -\xi w_n \pm w_n\sqrt{1-\xi^2}\, j$$

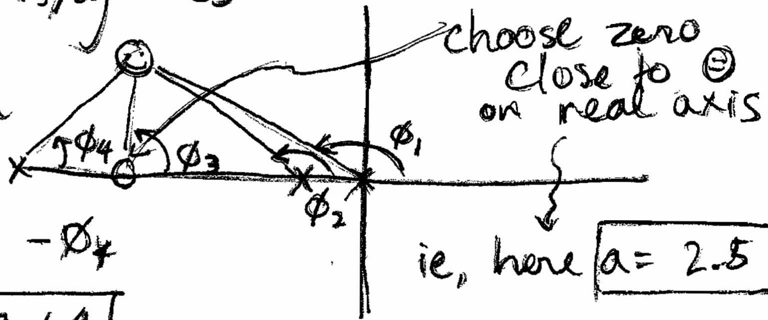
$\therefore$ CL poles must be at $s = -2.5 \pm \frac{5\sqrt{3}}{2} j$

Procedure:

① use lead to place CL poles @ $s = -2.5 \pm \frac{5\sqrt{3}}{2} j$

② use Lag to satisfy ess.

① $\boxed{\text{Lead} = K_L \dfrac{s+a}{s+b}}$, $b > a$



ie, here $\boxed{a = 2.5}$

$\angle G(s = ☺) = -\phi_1 - \phi_2 + \phi_3 - \phi_4$

$\Rightarrow \boxed{b = 8.64}$

$K_L = \left| \dfrac{1}{K(s = ☺)} \right| = \boxed{10.34}$

② $\boxed{\text{Lag} = \dfrac{s+z}{s+p}}$

$$e_{ss}(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \cdot \frac{1}{s^2}\left[\frac{1}{1+K(s)G(s)}\right]$$

$$e_{ss}(t) = \frac{0.5\,(8.64)\ p}{4(10.34)(2.5)\,z} \Rightarrow \frac{z}{p} \geq 2.09$$

ie if $\boxed{p = 0.01}$, $\boxed{z = 0.03}$, error is satisfied

3a. contd

$$\therefore \quad K(s) = 10.34 \frac{(s+2.5)}{(s+8.64)} \cdot \frac{(s+0.03)}{(s+0.01)}$$

3b) i) $y_{ss}(t) = |H(j1)| \sin(t + \measuredangle H(j1))$

ii) $y_{ss}(t) = |H(j2)| \sin(2t + \measuredangle H(j2))$

iii) $y_{ss}(t) = |H(j1)| \sin(t + \measuredangle H(j1))$
$+ |H(j2)| \sin(2t + \measuredangle H(j2))$

4. given the Bode diagram we predict the TF has break pts at

0.1, 10 in numerator
1, 100 in denominator

Also, there is a pole @ $j\omega = 0$

$(20 \log k) \approx -10\,dB$

$\Rightarrow k = 0.3162$

$$\Rightarrow KG(j\omega) = \frac{0.3162\,(10 j\omega + 1)(0.1 j\omega + 1)}{j\omega\,(j\omega + 1)(0.01 j\omega + 1)}$$