EE128 Problem Set 5 Solutions

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(1)

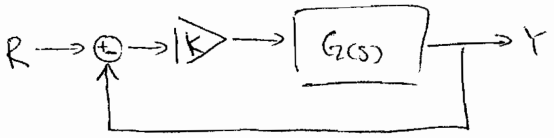


Figure 1: Unity feedback system

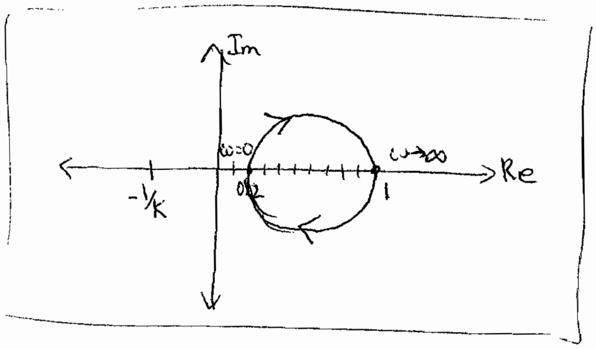
i) $G(s) = \frac{s+2}{s+10}$, $G(j\omega) = \frac{20+\omega^2+8j\omega}{100+\omega^2} \Rightarrow Re\{G(j\omega)\} = \frac{20+\omega^2}{100+\omega^2}$, $Im\{G(j\omega)\} = \frac{8\omega}{100+\omega^2}$

- Re-axis crossings at $\omega = 0$ and $\omega \to \infty$

  $Re\{G(0)\} = 0.2$, $Re\{G(j\infty)\} = 1$

- find direction of contour by intermediate value:

  $Im\{G(j1)\} = \frac{\oplus}{100+\oplus} > 0$, so contour travels clockwise for $\omega \in [0,\infty)$



3 encirclement regions: $-1/K < 0.2 \Rightarrow N = 0$

$0.2 < -1/K < 1 \Rightarrow N = 1$

$-1/K > 1 \Rightarrow N = 0$

because $P = 0$, want $N = 0$

so $-1/K < 0.2 \Rightarrow$ $K < -5$ or

and $-1/K > 1 \Rightarrow$ $K > -1$

ii) $G(s) = \frac{100(\frac{s}{10}+1)}{s(s-1)(\frac{s}{100}+1)} = \frac{1000\ (s+10)}{s(s-1)(s+100)}$, $G(j\omega) = -1000\ \frac{\omega^2(1090+\omega^2)+j\omega(89\omega^2-1000)}{\omega^6+10001\omega^4+10000\omega^2}$

$Re\{G(j\omega)\} = -1000\ \frac{1090+\omega^2}{\omega^4+10001\omega^2+10000}$, $Im\{G(j\omega)\} = \frac{-1000\ (89\omega^2-1000)}{\omega(\omega^4+10001\omega^2+10000)}$

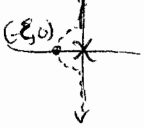
- Re-axis crossings at $\omega = \pm\sqrt{\frac{1000}{89}}$ and $\omega \to \infty$

  $Re\{G(j\sqrt{\frac{1000}{89}})\} = -8.99$, $\left.\begin{array}{l} Re\{G(j(+\infty))\} = 0^- \\ Im\{G(j(+\infty))\} = 0^- \end{array}\right\}$ approaches origin from Quadrant III

- as $\omega \to 0^+$, $Im\{G(j0^+)\} \Rightarrow +\infty$, $Re\{G(j0^+)\} \to \frac{-1000(1090)}{10000} \approx -109$

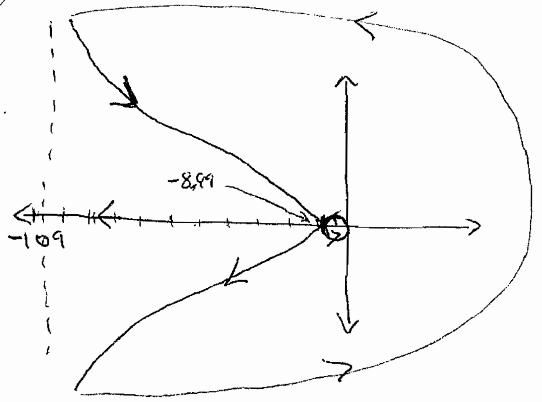
- which direction does contour close? (around $+\infty$ or $-\infty$)

  if we include pole @ 0 in contour:



  $G(s=-\varepsilon) = \frac{1000\ \oplus}{\ominus\ominus\oplus} = \oplus$

  so closes around $+\infty$



because we defined contour to include pole @ 0, $P = \underset{s=0}{1} + \underset{s=1}{1} = 2$

so we want $N = -2$

3 encirclement regions: $-1/K < -8.99 \Rightarrow N = 0$

$-8.99 < -1/K < 0 \Rightarrow N = -2$

$-1/K > 0 \Rightarrow N = -1$

$-1/K < 0 \Rightarrow K > 0$

$-1/K > -8.99 \Rightarrow \boxed{K > 0.111}$

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(2) Let $K = 1$ and $G(s) = \dfrac{1}{s^4(s+1)}$ Is this system stable?

$$G(j\omega) = \frac{1 - j\omega}{\omega^4(1+\omega^2)} \Rightarrow \text{Re}\{G(j\omega)\} = \frac{1}{\omega^4(1+\omega^2)}, \quad \text{Im}\{G(j\omega)\} = \frac{-\omega}{\omega^4(1+\omega^2)} = \frac{-1}{\omega^3(1+\omega^2)}$$

- asymptotic behavior

as $\omega \to 0^+$, $\text{Re}\{G(j0^+)\} \to +\infty$, $\text{Im}\{G(j0^+)\} \to -\infty$

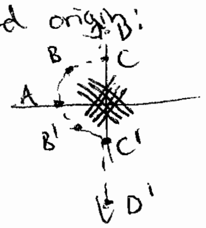
because order of denominator of Re > order of the denominator of Im, the Re component will approach ∞ faster than Im component

as $\omega \to +\infty$, $\text{Re}\{G(+j\infty)\} \to 0^+$, $\text{Im}\{G(+j\infty)\} \to 0^-$

as $\omega \to -\infty$, $\text{Re}\{G(-j\infty)\} \to 0^+$, $\text{Im}\{G(-j\infty)\} \to 0^+$

- analyze infinite behavior around origin:

↳ choose to include poles:

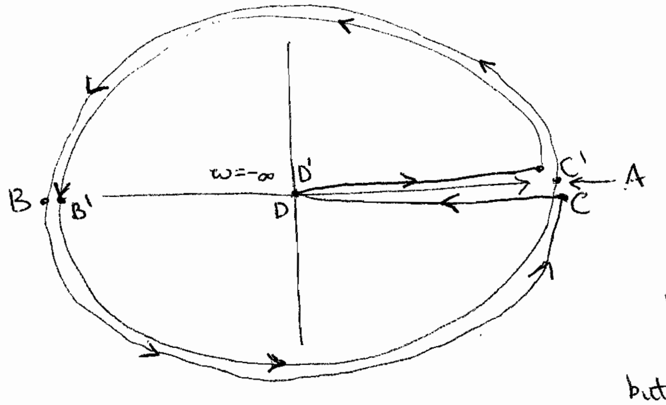


B is at 135°

assume D and D' are at $\pm\infty$

all points C', B', A, B, C will be at ∞, only difference is in the phase.

change in phase from C' to C is a $-180°$ phase change around 4 poles, so total phase change is $-4(-180) = 720°$, so 2 full encirclements of origin



included poles @ origin in contour, so $P = 4$

2 encirclement regions: $-1/K < 0 \Rightarrow N = -2$

$-1/K > 0 \Rightarrow N = -1$

$\boxed{\text{For } K=1, \text{ unstable}}$

$Z = -N - P < 0$
$= -(-2) - 4 = -2 \neq 0$

but we can also see that for all K, system will never be stable

(for easy check, try the root locus plot)

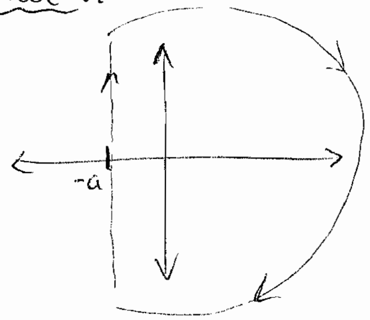
(3) The question is worded somewhat confusingly

Says to use Cauchy's Principle of the Argument to determine whether a closed loop system has poles to the left of $s=-a$.

So we need to redefine our contour in the s-plane.

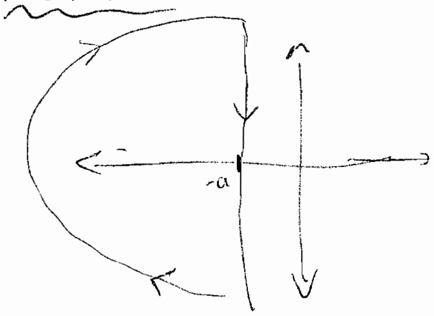
Draw the contour from $s=-a-\infty j$ to $s=-a+\infty j$ as a boundary and now we need to close it. You can close in either direction as long as you interpret the results correctly. Both ways I have arbitrarily drawn the contour clockwise.

Method 1:



This method will directly tell you if there are any poles to the right of $s=-a$, but indirectly ~~tells~~ ~~you~~ can tell you if all the poles are to the left of $s=-a$ (no poles to the right). This is similar to the actual Nyquist criterion and is more related to stability.

Method 2:



This method will directly tell you if there are any poles to the left of $s=-a$. In order to check for stability, you must take the additional step of counting all poles in your system.

Still interested in closed loop system, so check for encirclements of origin for $F(s)=1+KG(s)$ or equivalently encirclements of $-1/K$ in $KG(s)$.

$N_1=Z_1-P_1$, want $Z_1=0$ for stability, so look for $N_1=-P_1$ encirclements.

$N_2=Z_2-P_2$, want $Z_2=$ # of total poles in system $=n$

Look for $N_2=n-P_2$ encirclements

$G(s)=\frac{1}{s}$, $a=2$, $K=1$

I will use method 1.

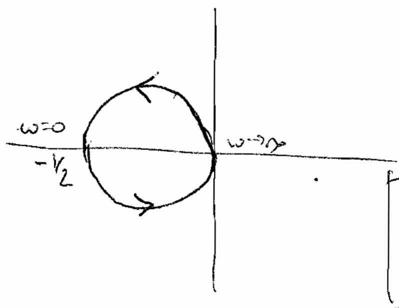
$G(j\omega)=\frac{1}{-2+j\omega}$

because contour is defined along $s=-a+j\omega$

$Re\{G(j\omega)\}=\frac{-2}{4+\omega^2}$, $Im\{G(j\omega)\}=\frac{-\omega}{4+\omega^2}$

as $\omega\to\infty$, $Im\to 0$; $Re\to 0^-$

as $\omega=0$, $Im=0$, $Re=-1/2$



OL pole to right of $s=-a$, so $P=1$

$N=0$ in ~~Nyq~~ Nyquist plot

So G(s) does NOT have a pole to the left of $s=-a$ at $K=1$