

# EECS 128 Introduction to Control Design Techniques

## Problem Set 6

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**Problem 1: Properties of the matrix exponential.** Show, by using a few terms of the infinite series expansion that:

- (a)  $e^{A(t+s)} = e^{At}e^{As}$ ;
- (b)  $e^{(A+B)t} = e^{At}e^{Bt}$  iff  $AB = BA$ ;
- (c)  $\frac{d}{dt}e^{At} = Ae^{At} = e^{At}A$ .

**Problem 2. Matrix review.** Let

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$$

What is  $A + D$ ?  $AD$ ?  $CA$ ?  $AB$ ?

**Problem 3. Compute  $e^{At}$ , where  $A$  is given below.**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

**Problem 4: Heat Exchanger.** Consider the simple heat exchanger shown in Figure 1, in which  $f_C$  and

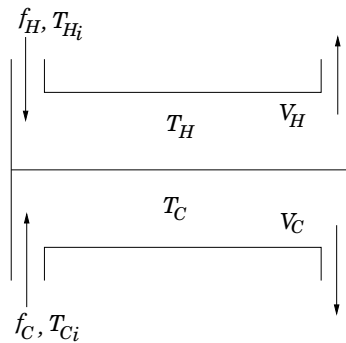


Figure 1: A simple heat exchange.

$f_H$  are the flows (assumed constant) of cold and hot water,  $T_H$  and  $T_C$  represent the temperatures in the hot and cold compartments, respectively,  $T_{Hi}$  and  $T_{Ci}$  denote the temperature of the hot and cold inflow, respectively, and  $V_H$  and  $V_C$  are the volumes of hot and cold water. The temperature in each compartment evolves according to:

$$V_C \frac{dT_C}{dt} = f_C(T_{Ci} - T_C) + \beta(T_H - T_C) \tag{1}$$

$$V_H \frac{dT_H}{dt} = f_H(T_{Hi} - T_H) - \beta(T_H - T_C) \tag{2}$$

Let the inputs to this system be  $u_1 = T_{Ci}$ ,  $u_2 = T_{Hi}$ , the outputs are  $y_1 = T_C$  and  $y_2 = T_H$ , and assume that  $f_C = f_H = 0.1$  ( $m^3/\text{min}$ ),  $\beta = 0.2$  ( $m^3/\text{min}$ ) and  $V_H = V_C = 1$  ( $m^3$ ). Assume that both compartments start at a temperature of 1 *units of temperature*.

(a) Write the state space and output equations for this system.

(b) In the absence of any input, determine  $y_1(t)$  and  $y_2(t)$ .

(c) Now, suppose  $T_{Hi} = 10$  and  $T_{Ci} = -10$  *units of temperature*. Determine  $y_1(t)$  and  $y_2(t)$ .

**Problem 5: Solution to state space systems.** Find the output response to a step input where the system is given by

$$\dot{X} = AX + BU \quad (3)$$

$$Y = CX + DU \quad (4)$$

where

$$X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = [1 \ 0] \quad D = 0 \quad (5)$$

and where

$$(a)A = \begin{bmatrix} 1 & 3 \\ 0 & 4 \end{bmatrix} \quad (b)A = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \quad (6)$$

For part (a) above, do the computation by hand and then check your result in MATLAB.