EECS 128 Introduction to Control Design Techniques
Problem Set 7

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Problem 1: Eigenvalue and Eigenvector review. Find the eigenvalues and eigenvectors (by hand) of the following matrix:

$$A = \begin{bmatrix} 5 & 2 & -2 \\ 1 & 7 & -1 \\ -3 & 0 & 6 \end{bmatrix} \tag{1}$$

Problem 2. Put the following system into modal canonical form.

$$\begin{aligned} \dot{X} &= \begin{bmatrix} -4 & 1 \\ -2 & -1 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U \\ y &= x_1 \end{aligned}$$

Problem 3: Modal form, and solution to state equations.

A system is described by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a & -a & 0 \\ 0 & 0 & 0 \\ 0 & -b & -c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where $a, b, c \in \Re$.

(a) Find the similarity transform which puts this system into modal form. Write the system representation in modal form.

(b) Under what conditions on $a, b, c \in \Re$ is this system BIBO stable?

(c) If $X(0) = [1\ 0\ 0]^T$ and if $U(t) = [1\ 0]^T$, find $X(t)$.

Problem 4. Preservation of Eigenvalues under Similarity Transform.

Consider a matrix $A \in \Re^{n \times n}$, and a non-singular matrix $P \in \Re^{n \times n}$. Show that the eigenvalues of $\overline{A} = PAP^{-1}$ are the same as those of A.

Remark: This important fact in linear algebra is the basis for the similarity transform: that a redefinition of the state (to a new set of state variables in which the equations above may have simpler representation) does not affect the stability of the system.

Problem 5: Asymptotic Internal Stability I.

Consider a diagonalizable matrix $A \in \Re^n$. Prove that the system $\dot{X} = AX$ is asymptotically internally stable if and only if the real parts of the eigenvalues of A are all less than zero.

Problem 6: Asymptotic Internal Stability II. Consider the system

$$\ddot{x} + d\dot{x} + kx = u \tag{2}$$

$$y = x \tag{3}$$

(a) Write the system equations in state space form.

(b) For what values of k and d is the system asymptotically stable?

Problem 7. A system's input output transfer function is

$$G(s) = \frac{1}{s^2(s+3)^3(s+1)}$$

Is the system asymptotically stable?