EECS 128 Introduction to Control Design Techniques
Problem Set 8

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Problem 1. Controllable and observable canonical forms. For the following transfer function, give state space descriptions in both controllable and observable canonical form:

$$\frac{Y(s)}{U(s)} = \frac{(s+10)(s^2+s+25)}{s^2(s+3)(s^2+s+36)}$$

Problem 2. Observable canonical form. Given

$$\frac{\partial^3 y}{\partial t^3} + 2\frac{\partial^2 y}{\partial t^2} + \frac{\partial y}{\partial t} + y = 3\frac{\partial^3 u}{\partial t^3} + 5\frac{\partial^2 u}{\partial t^2} + \frac{\partial u}{\partial t} + 2u \tag{1}$$

where $y(0) = 1, \dot{y}(0) = 0, \ddot{y}(0) = 2$, and $u(t) = 1 - e^{-5t}$. Find the observable canonical form with initial conditions.

Problem 3. Pole placement. Consider the dynamic system:

$$\frac{d^4\theta}{dt^4} + \alpha_1\frac{d^3\theta}{dt^3} + \alpha_2\frac{d^2\theta}{dt^2} + \alpha_3\frac{d\theta}{dt} + \alpha_4\theta = u$$

where u represents an input force, α_i are real scalars. Assuming that $\frac{d^3\theta}{dt^3}$, $\frac{d^2\theta}{dt^2}$, $\frac{d\theta}{dt}$, and θ can all be measured, design a state variable feedback control scheme which places the closed-loop eigenvalues at $s_1 = -1$, $s_2 = -1$, $s_3 = -1 + j1$, $s_4 = -1 - j1$.

Problem 4. State vs. Output Feedback.

Consider the plant described by:

$$\dot{X} = AX + Bu \tag{2}$$
$$y = CX \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ 7 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C = [1\ 3] \tag{4}$$

Find the closed loop characteristic equation if the feedback is: (a) $u = -[f_1\ f_2]X$, and (b) $u = -ky$.

Problem 5: Controllability. Show that the property of controllability (for a linear system) is preserved under similarity transformation.

Problem 6. Electrical systems with symmetry. Defining two state variables as the voltages across the two capacitors, is the system shown in Figure 1 controllable from v_i? Give an explanation (why or why not?) in terms of the circuit behavior. (HINT: Consider both the case in which the circuit is "balanced" ($C_1R_1 = C_2R_2$), and the case in which the circuit is not "balanced" ($C_1R_1 \neq C_2R_2$).

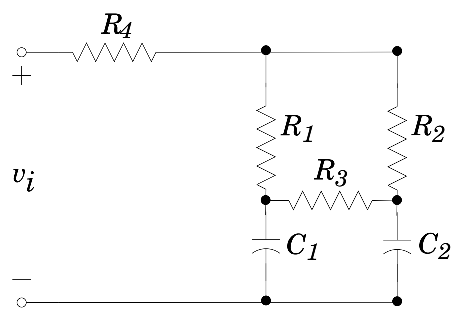

Figure 1: Electrical Bridge Network.