EE128 Problem Set 8 Solutions

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Problem 1. Controllable and observable canonical forms. For the following transfer function, give state space descriptions in both controllable and observable canonical form:

$$\frac{Y(s)}{U(s)} = \frac{(s+10)(s^2+s+25)}{s^2(s+3)(s^2+s+36)} = \frac{s^3+11s^2+35s+250}{s^5+4s^4+39s^3+108s^2}$$

So we have $a_0 = 0,\ a_1 = 0,\ a_2 = 108,\ a_3 = 39,\ a_4 = 4$ and $b_0 = 250,\ b_1 = 35,\ b_2 = 11,\ b_3 = 1$

Controllable canonical:

$$\text{Let}\quad \begin{aligned} x_1 &= x \\ x_2 &= \dot{x} \\ x_3 &= \ddot{x} \\ x_4 &= \dddot{x} \\ x_5 &= x^{(4)} \end{aligned}$$

$$\boxed{\begin{gathered} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -108 & -39 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \\ y = \begin{bmatrix} 250 & 35 & 11 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \end{gathered}}$$

Observable canonical:

We have $Y(s) = \frac{1}{s}(-4Y(s)) + \frac{1}{s^2}\big(1U(s) - 39Y(s)\big) + \frac{1}{s^3}\big(11U(s) - 108Y(s)\big) + \frac{1}{s^4}\big(35U(s)\big) + \frac{1}{s^5}\big(250U(s)\big)$

$$Y(s) = \frac{1}{s}\left(-4Y(s) + \frac{1}{s}\left(1U(s) - 39Y(s) + \frac{1}{s}\left(11U(s) - 108Y(s) + \frac{1}{s}\left(35U(s) + \frac{1}{s}\big(250U(s)\big)\right)\right)\right)\right)$$

$$\text{Let}\quad \begin{aligned} \dot{x}_5 &= 250u \\ \dot{x}_4 &= 35u + x_5 \\ \dot{x}_3 &= 11u - 108x_1 + x_4 \\ \dot{x}_2 &= u - 39x_1 + x_3 \\ \dot{x}_1 &= -4x_1 + x_2 \end{aligned}$$

$$\boxed{\begin{gathered} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \end{bmatrix} = \begin{bmatrix} -4 & 1 & 0 & 0 & 0 \\ -39 & 0 & 1 & 0 & 0 \\ -108 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 11 \\ 35 \\ 250 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u \end{gathered}}$$

Problem 2. Observable canonical form. Given

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 3\frac{d^3u}{dt^3} + 5\frac{d^2u}{dt^2} + \frac{du}{dt} + 2u$$

$y(0) = 1, \dot{y}(0) = 0, \ddot{y}(0) = 2, and\ u(t) = 1 - e^{-5t}$. Find the observable canonical form with initial conditions.

For TF, we assume zero initial conditions, so: $Y(s)(s^3 + 2s^2 + s + 1) = U(s)(3s^3 + 5s^2 + s + 2)$

$\frac{Y(s)}{U(s)} = \frac{3s^3+5s^2+s+2}{s^3+2s^2+s+1} = 3 + \frac{-s^2-2s-1}{s^3+2s^2+s+1}$. Set D = 3 and use remainder for A, B, and C:

Observable canonical:

$$Y(s) = \frac{1}{s}\big(-U(s) - 2Y(s)\big) + \frac{1}{s^2}\big(-2U(s) - Y(s)\big) + \frac{1}{s^3}\big(-U(s) - Y(s)\big)$$

$$Y(s) = \frac{1}{s}\left(-U(s) - 2Y(s) + \frac{1}{s}\left(-2U(s) - Y(s) + \frac{1}{s}\big(-U(s) - Y(s)\big)\right)\right)$$

Let
$$\begin{aligned} \dot{x}_3 &= -u - x_1 \\ \dot{x}_2 &= -2u - x_1 + x_3 \\ \dot{x}_1 &= -u - 2x_1 + x_2 \end{aligned}$$

$$\boxed{\begin{gathered} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 3 \end{bmatrix} u \end{gathered}}$$

Now use $y = x_1 + 3u$ with initial conditions:

We know that
$$\begin{array}{ccc} u(t) = 1 - e^{-5t} & \dot{u}(t) = 5e^{-5t} & \ddot{u}(t) = -25e^{-5t} \\ u(0) = 0 & \dot{u}(0) = 5 & \ddot{u}(0) = -25 \end{array}$$

$$y(0) = x_1(0) + 3u(0) = x_1(0) + 3(0) = x_1(0) = 1 \Rightarrow \boxed{x_1(0) = 1}$$

$$\dot{y}(0) = \dot{x}_1(0) + 3\dot{u}(0) = \big(-u(0) - 2x_1(0) + x_2(0)\big) + 3(5) = 0 - 2 + x_2(0) + 15 = 0 \Longrightarrow \boxed{x_2(0) = -13}$$

In above equation, $\dot{x}_1(0) = -15$

$$\begin{aligned} \ddot{y}(0) &= \ddot{x}_1(0) + 3\ddot{u}(0) = \big(-\dot{u}(0) - 2\dot{x}_1(0) + \dot{x}_2(0)\big) + 3(-25) \\ &= (-5 - 2(-15) + (-2u(0) - x_1(0) + x_3(0))) - 75 = 25 + \big(0 - 1 + x_3(0)\big) - 75 = 2 \ \Longrightarrow \ \boxed{x_3(0) = 53} \end{aligned}$$

$$\boxed{x(0) = \begin{bmatrix} 1 \\ -13 \\ 53 \end{bmatrix}}$$

Problem 3. Pole placement. Consider the dynamic system:

$$\frac{d^4\theta}{dt^4} + \alpha_1 \frac{d^3\theta}{dt^3} + \alpha_2 \frac{d^2\theta}{dt^2} + \alpha_3 \frac{d\theta}{dt} + \alpha_4 \theta = u$$

where u represents an input force, α_i are real scalars. Assuming that $\frac{d^3\theta}{dt^3}, \frac{d^2\theta}{dt^2}, \frac{d\theta}{dt}, and\ \theta$ can all be measured, design a state variable feedback control scheme which places the closed-loop eigenvalues at $s_1 = -1$, $s_2 = -1, s_3 = -1 + j1, s_4 = -1 - j1$.

Define $x_1 = \theta, x_2 = \dot{\theta}, x_3 = \ddot{\theta}, x_4 = \dddot{\theta}$. The system becomes: $\dot{x}_4 = u - \alpha_4 x_1 - \alpha_3 x_2 - \alpha_2 x_3 - \alpha_1 x_4$

In state space:
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha_4 & -\alpha_3 & -\alpha_2 & -\alpha_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

This is basically controllable canonical form. Use state feedback $u = -Fx$, where $F = [f_1\ f_2\ f_3\ f_4]$.

$\dot{x} = Ax + Bu = Ax - BFx = (A - BF)x$, so characteristic equation $\Delta(s) = \det\,(sI - (A - BF))$.

$$A - BF = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha_4 - f_1 & -\alpha_3 - f_2 & -\alpha_2 - f_3 & -\alpha_1 - f_4 \end{bmatrix}$$

$$sI - (A - BF) = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ \alpha_4 + f_1 & \alpha_3 + f_2 & \alpha_2 + f_3 & s + \alpha_1 + f_4 \end{bmatrix}$$

Expand along first column:

$$\det\big(sI - (A - BF)\big) = s\det\begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ \alpha_3 + f_2 & \alpha_2 + f_3 & s + \alpha_1 + f_4 \end{bmatrix} - (\alpha_4 + f_1)\det\begin{bmatrix} -1 & 0 & 0 \\ s & -1 & 0 \\ 0 & s & -1 \end{bmatrix}$$

Remark: Determinant of triangular matrices = product of diagonal entries

$$\det\big(sI - (A - BF)\big) = s\left(s\det\begin{bmatrix} s & -1 \\ \alpha_2 + f_3 & s + \alpha_1 + f_4 \end{bmatrix} + (\alpha_3 + f_2)(1)\right) - (\alpha_4 + f_1)(-1)$$

$$= s\Big(s\big(s^2 + (\alpha_1 + f_4)s + (\alpha_2 + f_3)\big) + (\alpha_3 + f_2)\Big) + (\alpha_4 + f_1)$$

$$= s\big(s^3 + (\alpha_1 + f_4)s^2 + (\alpha_2 + f_3)s + (\alpha_3 + f_2)\big) + (\alpha_4 + f_1)$$

$$= s^4 + (\alpha_1 + f_4)s^3 + (\alpha_2 + f_3)s^2 + (\alpha_3 + f_2)s + (\alpha_4 + f_1)$$

From desired poles:

$$\Delta(s) = (s+1)(s+1)(s^2 + 2s + 2) = (s^2 + 2s + 1)(s^2 + 2s + 2) = s^4 + 4s^3 + 7s^2 + 6s + 2$$

So we set: $\boxed{\begin{array}{l} f_1 = 2 - \alpha_4 \\ f_2 = 6 - \alpha_3 \\ f_3 = 7 - \alpha_2 \\ f_4 = 4 - \alpha_1 \end{array}}$

Problem 4. State vs. Output Feedback. Consider the plant described by:

$$\begin{aligned} \dot{X} &= AX + Bu \\ y &= CX \end{aligned} \qquad A = \begin{bmatrix} 0 & 1 \\ 7 & -4 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 2 \end{bmatrix},\ C = [1 \quad 3]$$

Find the closed loop characteristic equation if the feedback is: (a) $u = -[f_1\ f_2]X$, and (b) $u = -ky$.

(a) $u = -[f_1\ f_2]X$, so $\Delta(s) = \det\big(sI - (A - BF)\big) = \det\left(sI - \left(\begin{bmatrix} 0 & 1 \\ 7 & -4 \end{bmatrix} - \begin{bmatrix} f_1 & f_2 \\ 2f_1 & 2f_2 \end{bmatrix}\right)\right)$

$$\Delta(s) = \det\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -f_1 & 1 - f_2 \\ 7 - 2f_1 & -4 - 2f_2 \end{bmatrix}\right) = \det\begin{bmatrix} s + f_1 & f_2 - 1 \\ 2f_1 - 7 & s + 2f_2 + 4 \end{bmatrix}$$

$$\Delta(s) = (s + f_1)(s + 2f_2 + 4) - (f_2 - 1)(2f_1 - 7)$$

$$\Delta(s) = s^2 + (f_1 + 2f_2 + 4)s + (2f_1f_2 + 4f_1) - (2f_1f_2 - 2f_1 - 7f_2 + 7)$$

$$\boxed{\Delta(s) = s^2 + (f_1 + 2f_2 + 4)s + (6f_1 + 7f_2 - 7)}$$

(b) $u = -ky = -kCX$, so $\Delta(s) = \det\big(sI - (A - kBC)\big)$

Equating with part a, we see that $f_1 = k$ and $f_2 = 3k$.

$$\Delta(s) = s^2 + (f_1 + 2f_2 + 4)s + (6f_1 + 7f_2 - 7) = \boxed{s^2 + (7k + 4)s + (27k - 7) = \Delta(s)}$$

Problem 5: Controllability. Show that the property of controllability (for a linear system) is preserved under similarity transformation.

Use similarity transform T, so that we now have $\bar{X} = TX$, where $\exists T^{-1}$.

To get to our new state space equations, pre-multiply $\dot{X}$ equation by P and use the facts that $X = T^{-1}\bar{X}$ and $AX = AT^{-1}TX$:

$$\dot{\overline{X}} = TAT^{-1}\overline{X} + TBU$$

This means we have $\overline{A} = TAT^{-1}$ and $\overline{B} = TB$.

Now examine new controllability matrix: $\bar{\mathcal{C}} = [\bar{B} \;\; \bar{A}\bar{B} \;\; \ldots \;\; \bar{A}^{n-1}\bar{B}]$.

We can show that $\bar{A}^i\bar{B} = (TAT^{-1})^i(TB) = TA^iB$ for all $i \geq 0$ by cancellation of inner Ts and T^{-1}s.

So now $\bar{\mathcal{C}} = [TB \;\; TAB \;\; TA^2B \;\; \ldots \;\; TA^{n-1}B]$.

Pull out pre-multiplied T to get $\bar{\mathcal{C}} = T\,[B \;\; AB \;\; A^2B \;\; \ldots \;\; A^{n-1}B] = T\mathcal{C}$.

Because T is an invertible (and thus, full-rank) matrix, multiplication of T does NOT affect the rank. So if $\mathcal{C}$ is full rank (and thus controllable), then so is $\bar{\mathcal{C}}$.

Therefore controllability is preserved under a similarity transform.

Problem 6. Electrical systems with symmetry. Defining two state variables as the voltages across the two capacitors, is the system shown in Figure 1 controllable from v_i? Give an explanation (why or why not?) in terms of the circuit behavior. (HINT: Consider both the case in which the circuit is "balanced" ($C_1R_1 = C_2R_2$), and the case in which the circuit is not "balanced" ($C_1R_1 \neq C_2R_2$).

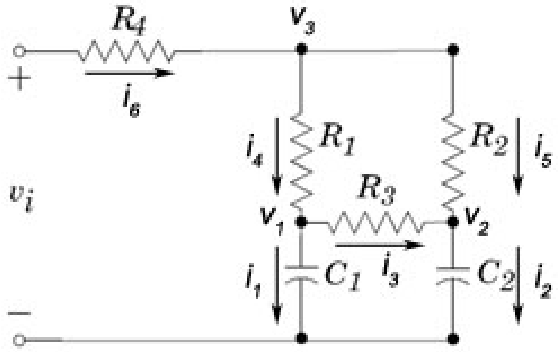

Figure 1: Electrical Bridge Network

See figure 1 for the definition of currents i_1 to i_6 and voltages v_1 to v_3.

$i_1 = C_1\frac{dv_1}{dt},\; i_2 = C_2\frac{dv_2}{dt},\; i_3 = \frac{v_1-v_2}{R_3},\; i_4 = \frac{v_3-v_1}{R_1},\; i_5 = \frac{v_3-v_2}{R_2},\; i_6 = \frac{v_i-v_3}{R_4}$

KCL on top node: $i_6 = i_4 + i_5 \;\Longrightarrow\; \frac{v_i-v_3}{R_4} = \frac{v_3-v_1}{R_1} + \frac{v_3-v_2}{R_2} \;\Longrightarrow\; \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right)v_3 = \frac{v_i}{R_4}+\frac{v_1}{R_1}+\frac{v_2}{R_2}$

$v_3 = \frac{R_1R_2v_i+R_2R_4v_1+R_1R_4v_2}{R_1R_2+R_1R_4+R_2R_4}$, for simplification, use $a = R_1R_2 + R_1R_4 + R_2R_4$.

KCL on v$_1$ node: $i_4 = i_1 + i_3 \;\Longrightarrow\; \frac{v_3-v_1}{R_1} = C_1\frac{dv_1}{dt} + \frac{v_1-v_2}{R_3} \;\Longrightarrow\; C_1\frac{dv_1}{dt} = \frac{v_3}{R_1} - \left(\frac{1}{R_1}+\frac{1}{R_3}\right)v_1 + \frac{v_2}{R_3}$

Substitute v$_3$: $C_1\frac{dv_1}{dt} = \frac{R_2}{a}v_i + \left(\frac{R_2R_4}{R_1a} - \frac{1}{R_1} - \frac{1}{R_3}\right)v_1 + \left(\frac{R_4}{a}+\frac{1}{R_3}\right)v_2 \qquad (1)$

KCL on v$_2$ node: $i_5 + i_3 = i_2 \implies \frac{v_3 - v_2}{R_2} + \frac{v_1 - v_2}{R_3} = C_2 \frac{dv_2}{dt} \implies C_2 \frac{dv_2}{dt} = \frac{v_3}{R_2} + \frac{v_1}{R_3} - \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_2$

Substitute v$_3$: $C_2 \frac{dv_2}{dt} = \frac{R_1}{a} v_i + \left(\frac{R_4}{a} + \frac{1}{R_3}\right) v_1 + \left(\frac{R_1 R_4}{R_2 a} - \frac{1}{R_2} - \frac{1}{R_3}\right) v_2$ (2)

From equations (1) and (2), we can build a state space model:

$$\begin{bmatrix} \dot{v_1} \\ \dot{v_2} \end{bmatrix} = \begin{bmatrix} \frac{1}{C_1}\left(\frac{R_2 R_4}{R_1 a} - \frac{1}{R_1} - \frac{1}{R_3}\right) & \frac{1}{C_1}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) \\ \frac{1}{C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) & \frac{1}{C_2}\left(\frac{R_1 R_4}{R_2 a} - \frac{1}{R_2} - \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} \frac{R_2}{aC_1} \\ \frac{R_1}{aC_2} \end{bmatrix} v_i$$

Now we check for controllability using the controllability matrix:

$$AB = \begin{bmatrix} \frac{R_2}{aC_1{}^2}\left(\frac{R_2 R_4}{R_1 a} - \frac{1}{R_1} - \frac{1}{R_3}\right) + \frac{R_1}{aC_1 C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) \\ \frac{R_2}{aC_1 C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) + \frac{R_1}{aC_2{}^2}\left(\frac{R_1 R_4}{R_2 a} - \frac{1}{R_2} - \frac{1}{R_3}\right) \end{bmatrix}$$

$$\mathcal{C} = [B \quad AB] = \begin{bmatrix} \frac{R_2}{aC_1} & \frac{R_2}{aC_1{}^2}\left(\frac{R_2 R_4}{R_1 a} - \frac{1}{R_1} - \frac{1}{R_3}\right) + \frac{R_1}{aC_1 C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) \\ \frac{R_1}{aC_2} & \frac{R_2}{aC_1 C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) + \frac{R_1}{aC_2{}^2}\left(\frac{R_1 R_4}{R_2 a} - \frac{1}{R_2} - \frac{1}{R_3}\right) \end{bmatrix}$$

$$\det(\mathcal{C}) = \frac{R_2{}^2}{a^2 C_1{}^2 C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) + \frac{R_1 R_2}{a^2 C_1 C_2{}^2}\left(\frac{R_1 R_4}{R_2 a} - \frac{1}{R_2} - \frac{1}{R_3}\right) - \frac{R_1 R_2}{a^2 C_1{}^2 C_2}\left(\frac{R_2 R_4}{R_1 a} - \frac{1}{R_1} - \frac{1}{R_3}\right) - \frac{R_1{}^2}{a^2 C_1 C_2{}^2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right)$$

$$\det(\mathcal{C}) = \frac{1}{a^2 C_1 C_2}\left[\frac{R_2{}^2}{C_1}\left(\frac{R_4}{a} + \frac{1}{R_3}\right) + \frac{R_1 R_2}{C_2}\left(\frac{R_1 R_4}{R_2 a} - \frac{1}{R_2} - \frac{1}{R_3}\right) - \frac{R_1 R_2}{C_1}\left(\frac{R_2 R_4}{R_1 a} - \frac{1}{R_1} - \frac{1}{R_3}\right) - \frac{R_1{}^2}{C_2}\left(\frac{R_4}{a} + \frac{1}{R_3}\right)\right]$$

$$\det(\mathcal{C}) = \frac{1}{a^2 C_1 C_2}\left[\left(\frac{R_2{}^2}{C_1} - \frac{R_1{}^2}{C_2}\right)\left(\frac{R_4}{a} + \frac{1}{R_3}\right) + \frac{R_1{}^2 R_4}{aC_2} - \frac{R_1}{C_2} - \frac{R_1 R_2}{R_3 C_2} - \frac{R_2{}^2 R_4}{R_1 a} + \frac{R_2}{C_1} + \frac{R_1 R_2}{R_3 C_1}\right]$$

$$\det(\mathcal{C}) = \frac{1}{a^2 C_1 C_2}\left[\left(\frac{R_2{}^2}{R_3 C_1} - \frac{R_1{}^2}{R_3 C_2}\right) + \frac{R_1 R_2}{R_3 C_1} - \frac{R_1 R_2}{R_3 C_2} + \left(\frac{R_2}{C_1} - \frac{R_1}{C_2}\right)\right]$$

$$\det(\mathcal{C}) = \frac{1}{a^2 C_1 C_2}\left[\frac{1}{R_3}\left(\frac{R_2(R_1+R_2)}{C_1} - \frac{R_1(R_1+R_2)}{C_2}\right) + \left(\frac{R_2}{C_1} - \frac{R_1}{C_2}\right)\right]$$

$$\det(\mathcal{C}) = \frac{1}{a^2 C_1 C_2}\left[\left(\frac{(R_1+R_2)}{R_3} + 1\right)\left(\frac{R_2}{C_1} - \frac{R_1}{C_2}\right)\right] = \frac{R_1+R_2+R_3}{a^2 R_3 C_1 C_2}\left(\frac{R_2}{C_1} - \frac{R_1}{C_2}\right)$$

$$\boxed{\det(\mathcal{C}) = \frac{R_1 + R_2 + R_3}{(R_1 R_2 + R_1 R_4 + R_2 R_4)^2 R_3 C_1 C_2}\left(\frac{R_2}{C_1} - \frac{R_1}{C_2}\right)}$$

In order for system to be controllable, we need $\det(\mathcal{C}) \neq 0$.

Assuming non-negative values for all resistors and capacitors, $\det(\mathcal{C}) = 0$ when $\frac{R_2}{C_1} = \frac{R_1}{C_2}$, so when $\boxed{R_1 C_1 = R_2 C_2}$. System will be uncontrollable under this condition.

(Or when $\boxed{C_1 \to \infty,\ C_2 \to \infty,\ or\ R_3 = \infty}$ – these are ungraded cases)

Beyond these equations, it should be intuitive from looking at the circuit diagram that the system will be uncontrollable if $\boxed{R_3 = 0}$. In this case, $v_1 = v_2$ and then there'd be no way to decouple the two state variables.