EECS 128 Introduction to Control Design Techniques
Problem Set 9

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Problem 1: Controllability and observability.

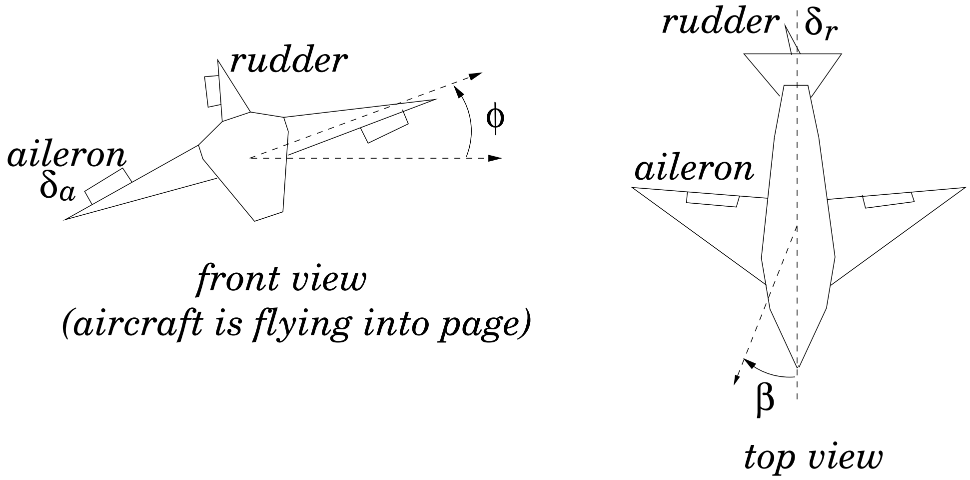

Figure 1: Lateral axes of an aircraft.

An approximate linear model of the lateral dynamics of an aircraft, for a particular set of flight conditions, has the linearized state and control vectors:

$$x = \begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{1}$$

where p and r are incremental roll and yaw rates, β is an incremental sideslip angle, and ϕ is an incremental roll angle. The control inputs are the incremental changes in the aileron angle δ_a and in the rudder angle δ_r, respectively. These variables are shown in Figure 1. The state space equation for this model is $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} -10 & 0 & -10 & 0 \\ 0 & -0.7 & 9 & 0 \\ 0 & -1 & -0.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

(a) Suppose a malfunction prevents manipulation of the input δ_r. Is it possible to completely control the aircraft using only δ_a?

(b) If you had your choice of *only one* of the following sensors, which would you choose? Explain.
(i) A rate gyro which measures the roll rate p.
(ii) A bank indicator which measures ϕ.

Problem 2: Observer design.

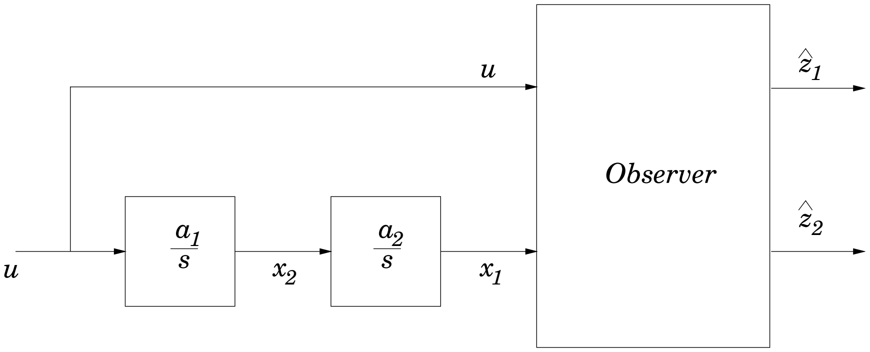

Figure 2: Simple model of a DC Servo system.

Figure 2 shows a block diagram representation of a simple model of a DC servo system: x_1 is a voltage signal proportional to the output angular velocity x_2 (ie. a tacho signal is not available).

(a) Design a full order observer, with observer gain matrix T given by

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \tag{3}$$

for x_1 and x_2 so that the characteristic polynomial associated with the error dynamics is given by:

$$\Delta_e(s) = s^2 + 2\zeta_e\omega_e s + \omega_e^2 \tag{4}$$

("Design" means write the equations for the observer, with expressions for gains T_1 and T_2.)

(b) Now, the observer is a system with inputs u and x_1, and outputs $\hat{z_1}$ and $\hat{z_2}$. Thus, there are four possible transfer functions between inputs and outputs – these may be included as elements in a 2×2 matrix. Evaluate the following *matrix of transfer functions* $M(s)$ between the inputs to the observer u and x_1, and its outputs $\hat{z_1}$ and $\hat{z_2}$:

$$M(s) = \begin{bmatrix} \hat{z}_1(s)/u(s) & \hat{z}_1(s)/x_1(s) \\ \hat{z}_2(s)/u(s) & \hat{z}_2(s)/x_1(s) \end{bmatrix} \tag{5}$$

as a function of gains T_1 and T_2, as well as system parameters a_1 and a_2.

(c) Now determine $M(s)$ as $T_2 \to \infty$. Discuss the meaning of the result.

Problem 3: Observer design.

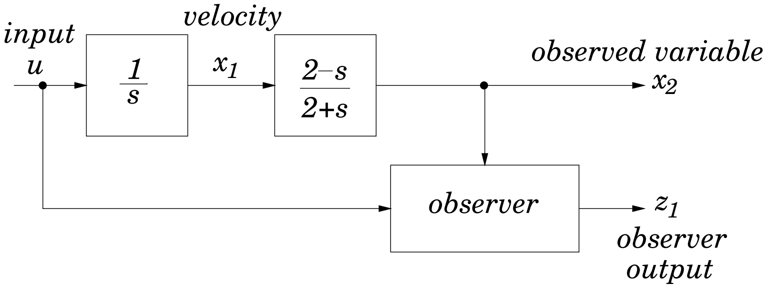

Figure 3: Velocity Observation System.

Figure 3 shows a velocity observation system where x_1 is the velocity to be observed. An observer is to be constructed to track x_1, using u and x_2 as inputs. The variable x_2 is obtained from x_1 through a sensor having the known transfer function

$$\frac{2-s}{2+s} \tag{6}$$

as shown in Figure 3.

(a) Derive a set of state-space equations for the system with state variables x_1 and x_2, input u and output x_2. Check your result by calculating the transfer function.

(b) Design an observer with states z_1 and z_2 to track x_1 and x_2 respectively. Choose both observer eigenvalues to be at -4. Write out the state space equations for the observer.

(c) Derive the combined state equation for the system plus observer. Take as state variables x_1, x_2, $e_1 = x_1 - z_1$, and $e_2 = x_2 - z_2$. Take u as input and z_1 as the output. Is this system controllable and/or observable? Give physical reasons for any states being uncontrollable or unobservable.

(d) What is the transfer function relating u to z_1? Explain your result.

Problem 4: Observer-controller for a nonlinear system.

The simplified dynamics of a magnetically suspended steel ball are given by:

$$m\ddot{y} = mg - c\frac{u^2}{y^2} \tag{7}$$

where the input u represents the current supplied to the electromagnet, y is the vertical position of the ball, which may be measured by a position sensor, g is gravitational acceleration, m is the mass of the ball, and c is a positive constant such that the force on the ball due to the electromagnet is $c\frac{u^2}{y^2}$. Assume a normalization such that $m = g = c = 1$.

(a) Using the states $x_1 = y$ and $x_2 = \dot{y}$ write down a nonlinear state space description of this system.

(b) What *equilibrium control input* u_e must be applied to suspend the ball at $y = 1$ m?

(c) Write the linearized state space equations for state and input variables representing perturbations away from the equilibrium of part (b).

(d) Is the linearized model stable? What can you conclude about the stability of the nonlinear system close to the equilibrium point x_e?

(e) Is the linearized model controllable? Observable?

(f) Design a state feedback controller for the linearized system, to place the closed loop poles at $-1, -1$.

(g) Design a full order observer, so that the state estimate error dynamics has eigenvalues at $-5, -5$.

(h) Combine your answers above to find an output feedback controller $K(s)$ for the linearized system that places the closed loop system poles at $-1, -1, -5, -5$.

(i) Now, suppose that you applied this controller to the original nonlinear system; discuss how you would expect the system to behave. How would the behavior change if you had chosen controller poles at $-5, -5$, and observer poles at $-20, -20$?

Problem 5: Linear Quadratic Optimization.

Consider an object of mass $m = 1$ moving along the x-axis in response to a force input $u(t)$. The object's dynamics can be described simply as $\ddot{x} = u(t)$. Suppose you would like to design an input $u(t)$ which will move the object from any initial position and velocity, to come to rest at the position $x = 0$. Using the linear quadratic regulator discussed in class, formulate an appropriate quadratic cost functional, and solve the problem in MATLAB, showing simulations of your results for different weightings on the state and input.