Lab 3: Model-based Position Control of a Cart

I. Objective

The goal of this lab is to help understand the methodology to design a controller using the given plant dynamics. Specifically, we would do position control of a cart by developing various controllers and then comparing their performances.

II. Software and equipment

1. Computer with MATLAB, Simulink, RTW, and QuaRC installed (204-04, 204-06, 204-12).
2. ee128 student account

III. Theory

1. Plant Dynamics

Since the plant here, a motor with a cart, is the same as Lab 2, we just provide a summary of its dynamics below. Please refer to Lab 2 for the derivation.

$$(m_c r^2 R_m)\ddot{x} + \left(K_m^2 K_g^2\right)\dot{x} = \left(r K_m K_g\right)V \quad (1)$$

In the equation above:
$\boldsymbol{V}$ is the input voltage (volts)
$\boldsymbol{m_c}$ is the mass of the car (kilograms)
$\boldsymbol{r}$ is the radius of the motor gear (meters)
$\boldsymbol{R_m}$ is resistance of the motor windings (ohms)
$\boldsymbol{K_m}$ is the back EMF constant $\left(\frac{V}{rad/sec}\right)$
$\boldsymbol{K_g}$ is the motorbox gear ratio (no units)

The motor-cart system model is a second order model, whose dynamics we now examine:

2. Second order dynamics

A second order linear system is described by the general differential equation of the form:

$$\ddot{y} + 2\xi\omega_n\dot{y} + {\omega_n}^2 y = bu(t) \quad (2)$$

The above equation when expressed in the Laplace domain becomes:

$$\frac{Y(s)}{U(s)} = \frac{b}{s^2 + 2\xi\omega_n s + {\omega_n}^2} \quad (3)$$

For $\xi \leq 1$, we find that above system has complex poles which are depicted on a graph in the figure below:

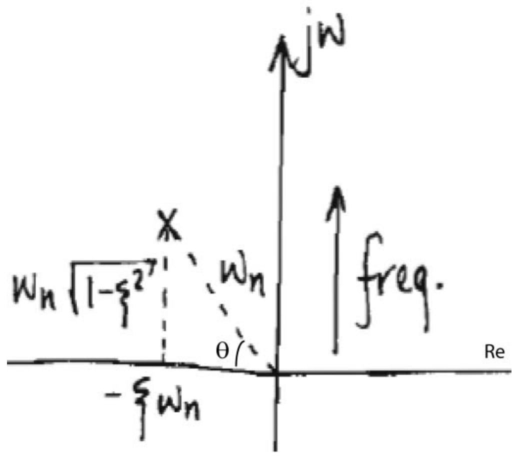

Figure 1: Location of a pole in terms of $\boldsymbol{\omega_n}$ and $\boldsymbol{\xi}$.

The parameter ω_n is called the natural frequency and is a measure of the speed of the response of the second order system while ξ is a measure of the damping in the system.

For complex poles in the left half plane, we make the following important observations from the figure above:

- The length of the complex vector from the origin to the pole is ω_n
- The cosine of the angle of the vector with the negative real axis equals ξ i.e. $\cos\theta = \xi$ (notice that this is NOT the same θ as defined in the lecture notes!)

A typical step response for a second order system is as shown in figure below:

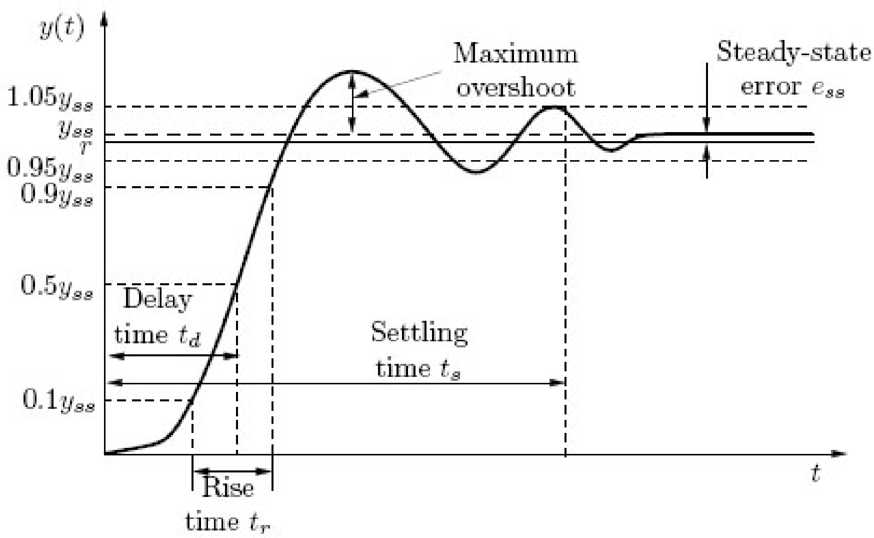

Figure 2: Typical step response of a control system.

Two important performance metrics for second order systems are their rise time and maximum overshoot. In terms of parameters ω_n & ξ they are given as:

$$t_r \approx \frac{0.8+2.5\xi}{\omega_n} \tag{5}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \tag{6}$$

NOTE:
In order to get faster response (small t_r) and smaller overshoot (small M_p) we would like the closed loop poles to have large radial distance (leading to a large ω_n) and small angle θ (leading to a higher ξ).

3. Step Response of a System

Please review the Theory section of Lab 2 for definitions of steady-state error, maximum overshoot, delay time, rise time, and settling time.

IV. Prelab

1. Position Controller Design

We set the following performance objective to be achieved by the feedback system for cart position control:

1. Rise time $t_r \leq 0.15\ s$
2. Maximum overshoot $M_p \leq 5\%$

The objective is to design a feedback system that will help us achieve these desired performance specifications. The general guidelines to proceed with the design are outlined below:

Step 1. Plant Model

Use your corrected state space or transfer function representation of the system from Lab 2. The variable values are restated below:

$\boldsymbol{m_c}$ = 0.57 + 0.37 = 0.94 kg
$\boldsymbol{r}$ = 0.00635 m
$\boldsymbol{R_m}$ = 2.6 Ω
$\boldsymbol{K_m}$ = 0.00767 V*s/rad
$\boldsymbol{K_g}$ = 3.71

Determine the poles of this transfer function. Is the plant BIBO stable?

Step 2: Design parameters

Using the given desired performance objectives and equations (5) and (6), come up with desired values of ω_n and ξ.

Step 3: Proportional Controller

The first controller that we will try is a proportional controller. Build a Simulink model in a simple negative feedback with a constant gain K. Vary the value of K, for simplicity over the range 30-60.

- As K increases, what happens to the rise time and overshoot?
- With just a P controller, can the desired performance specifications be achieved?

Put any 5 of these plots superimposed on each other in your report. Get the K value for which at least the rise time performance specification is met. This value will be used in the lab during experimentation.

The above observations can also be made from the root locus plot. Plot the root locus for the plant transfer function and make the following observations:

- For complex poles of the closed loop transfer function, as K increases what happens to the radial distance and the angle θ?
- Going back to Figure 1 and equations (5) and (6), what does this mean will happen to ξ and ω_n as K increases?

Note that this is not the desired behavior we stated in the enclosed note above.

Step 4: PD Controller

In order to meet the design constraints, we will need derivative action which will help "apply the brakes earlier."

In order to reduce the overshoot we use derivative action in conjunction with proportional action. Its form is as follows:

$$k(t) = k_P\left(x_{ref} - x\right) + k_D\left(\dot{x}_{ref} - \dot{x}\right) = k_P e(t) + k_D \dot{e}(t) \qquad (7)$$

Put this in the plant dynamics equation from Step 1 and obtain the transfer function from x_{ref} to x. Observe that a PD controller introduces a zero in the plant transfer function at $s = -\frac{k_P}{k_D}$.

For further analysis we assume that we will place this zero at $s = -13$. This location of the zero can be found through design techniques which will be covered in subsequent labs. This changes the form of the PD controller to $K(s) = K(s + 13)$. Plot the root locus for this transfer function of the "modified" plant which includes the zero. By comparing this root locus with the one plotted in Step 3 make the following observations:

- A left half plane zero tends to pull the root locus towards it.
- There exists a portion of the root locus which has the following properties
  - i. Has complex closed loop poles and
  - ii. ω_n and ξ both increase as K increases

This is what we desire as stated in the enclosed note above.

Determine a K value from this root locus now for which the performance specifications are met and thus determine k_P and k_D for the controller. Starting with your calculated values of k_P and k_D, increase k_D by some regular interval and display on the same plot.

- As k_D increases, what happens to the rise time and overshoot?

V. Lab:

1. Proportional Control

Experiment with the proportional controller trying out various gain values from 0 to 60 and observe the variation of rise time with gain value. Ignoring the overshoot, can the rise time performance be met? If so, give the K value and system response plot. If not, can you explain why this might be the case?

2. PD Control

Now, put in the PD controller that was designed in step 4 of the prelab and verify if the desired performance specifications are met. If they are not, then do an "intelligent" tuning of the gain values around the found nominal values so that the required specifications are met.

The lab instructions for operation of the hardware remain the same as for Lab 2.

3. Extra Credit (1 pt each)

Using your TUNED PD controller from part 2, what is the performance like for the following input signals? MAKE SURE YOUR END TIME IS FINITE AND ≤ 20. (in case something goes wrong)

- Pulse generator (50% pulse width, amplitude ≤ 4, period ≥ 3)
- Sine wave (amplitude ≤ 4, frequency ≤ 4)
- Exponentially-DECAYING sine wave (initial amplitude ≤ 6, pick a reasonable time constant so that the decay is evident, but not too sudden or gradual

Include your modified Simulink/QuaRC model as well as plots of the input signal and the actual hardware output (separate or superimposed).

VI. Revision History

Semester and Revision	Author(s)	Comments
Fall 2008 Rev. 1.1	Justin Hsia	Lab formatting, extra credit
Fall 2008 Rev. 1.0	Pranav Shah	Initial version of lab write-up