Lab 5b: Luenberger Observer Design for Inverted Pendulum

I. Purpose

The objective of this lab is to design a Luenberger state estimator to estimate the state of an inverted pendulum system given position of the cart and the pendulum and utilize this estimate for feedback control of the system.

II. Theory

Pole placement design is performed under the assumption that all states of the system are measurable. However, in physical systems not all states may be measurable and thus states need to be estimated based on limited sensing available.

The Luenberger observer uses the architecture shown below for state estimation.

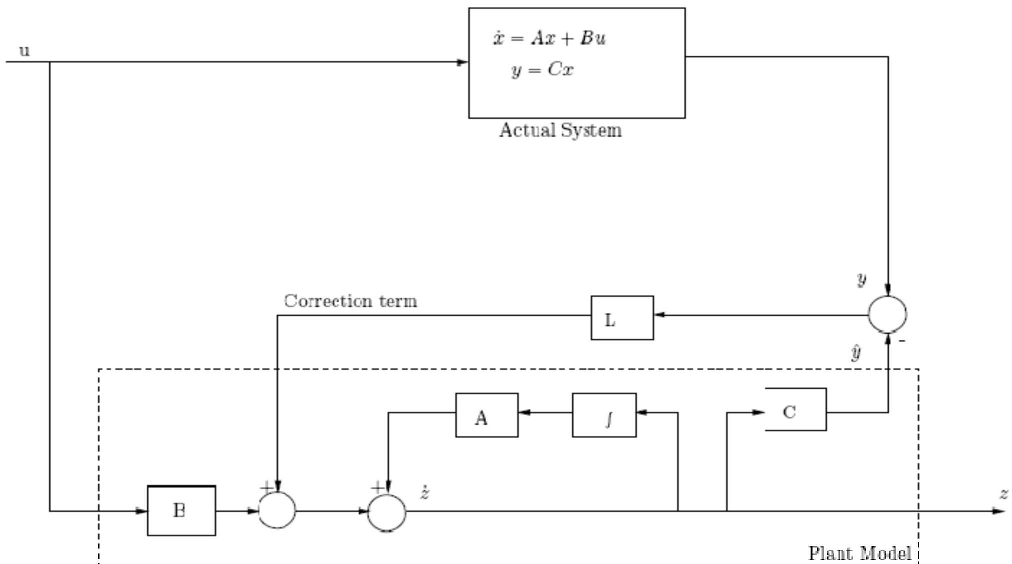

Figure 1: Luenberger observer architecture

As discussed in class, the dynamics of the observer can be stated as:

$$\dot{z} = \underbrace{Az + Bu}_{Predictor} + \underbrace{L(y - \hat{y})}_{Corrector} \tag{1}$$

The predictor part of the above equation is a replica of the plant dynamics. However, because of uncertainties in the plant model, estimate of the state using only the predictor will never match the actual state of the system. A correction term is thus needed and this forms the Luenberger observer. The correction term corrects future estimates of the state based on the present error in estimation.

The gain L can be considered as a parameter which weighs the relative importance between the predictor and the corrector in state estimation. Intuitively, it can be seen that a "low" value for L is chosen when our confidence in the model (i.e. the predictor) is high and/or confidence in measurement y is low (i.e. when the measurements are noisy) and vice-versa for a "high" L.

The objective of this lab is to design the gain L and use the state estimator for feedback control of the inverted-pendulum system.

III. Prelab

(a) The model for inverted-pendulum system and the desired position of the closed loop poles ($\mathbf{-1.7 \pm 9j}$ and $\mathbf{-2 \pm 1.6j}$) remain the same as in the previous lab.

The gain L is chosen such that the matrix $A - LC$ has eigenvalues in the left half plane. Further, the exact eigenvalues of $A - LC$ govern the rate at which the state estimate(z) converges to the actual estimate(x) of the system. It is normally desired that the observer estimate of the state converges to the actual state at least an order of magnitude faster than the performance desired of the system. This helps the controller in obtaining a "good" estimate of the actual state of the system in relatively short time and thus it can take appropriate control action.

For this lab, we desire to place the eigenvalues of the observer at $\mathbf{-10 \pm 15j}$ and $\mathbf{-12 \pm 17j}$. Note that they have been chosen to be relatively "away" from the desired closed loop poles. Using MATLAB, choose a L such that this is achieved.

(b) Implement the above designed observer in MATLAB. The implementation of the observer would be similar to that shown in figure 1 but with a model of the system in place of the actual system. Utilize the estimate(z) of the state for state feedback implementation. You may use the gain matrix designed in the previous lab since the desired location of the closed loop poles have not been changed.

Perform simulation of the system for unit perturbation of the plant. You may do this by giving initial conditions to the state vector of the plant model in the Simulink. Plot the output response of the plant with time(y) i.e. position of both the cart and pendulum and see how well the system performs.

Also, plot the observer estimate(z) of the state along with the actual state of the plant(x), which may be obtained from the plant model in Simulink. However, note that this cannot be done with the physical plant as we have no measurement of the actual state(x). Observe how error in estimation($e = z - y$) varies with time.

IV. Lab

(a) Implement the state feedback controller along with the Luenberger observer in the lab.

(b) Observe and record the output of the observer($\hat{y}$) and the actual measurement(y) i.e both the position of the cart and the pendulum on the same plot. The difference between these two signals indicates how well the observer estimates the state of the system.

(c) Now implement the state feedback controller in the manner in which it was done in the previous lab (just run your old model). Remember that the closed loop poles are the same with the both these systems. Do you physically see any noticeable difference in performance? Observe and record a time trace of the measurements made of the cart position and the angular position of the rod.

(d) Using time traces from steps (b) and (c), compare the tracking performance obtained with these two controllers. Comment on the difference, if any, in the tracking performance and attribute a reason to it. (**Hint:** Compare the estimate of the velocity of the cart and the pendulum with that obtained by taking derivative from the position signals of cart and the pendulum that we used in the previous lab.)

V. Optional Question (need not be part of your report)

Analytically, is there any difference between taking the derivative of the position signals of the cart and pendulum compared to using a Luenberger observer for estimation? A suggestion is to look at how these two schemes differ when a noise is present in the actual measurement of the positions.

V. Revision History

Semester and Revision	Author(s)	Comments
Fall 2008 Rev. 1.0	Justin Hsia	Converted lab to Word
Fall 2008 Rev. 1.0	Pranav Shah	Initial lab write-up