EECS 128 LECTURE NOTES 4

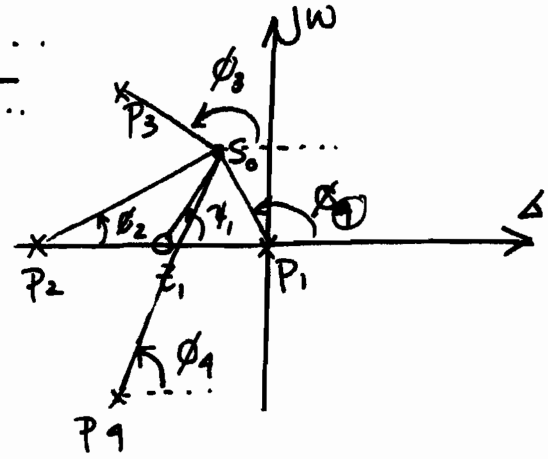
GOALS:

- review Bode Plots
- discuss relationship between Bode plot (of open loop dynamics) and Stability and performance of Closed loop system.
  - gain margin
  - phase margin
  - crossover freq. vs bandwidth
- Some examples

REFS: § 6.1, 6.2, 6.4, 6.6

Evaluation of $G(s)$ at $s = s_o$:

$$G(s) = K\,\frac{(s-z_1)(s-z_2)\cdots}{(s-p_1)(s-p_2)\cdots}$$



Find:

$$|G(s_o)| = K\,\frac{|s_o-z_1|\,|s_o-z_2|\cdots}{|s_o-p_1|\,|s_o-p_2|\cdots}$$

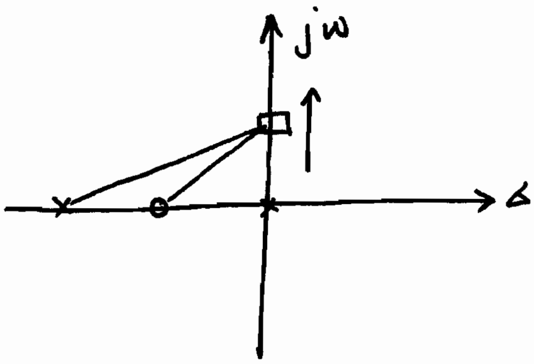
length of vectors

$\measuredangle G(s_o) = \psi_1 - \phi_1 - \phi_2 - \phi_3 - \phi_4 \cdots$ angles ccw from positive real line

. What will happen if $s_o \to z_1$? $|G(s_o)| \to 0$

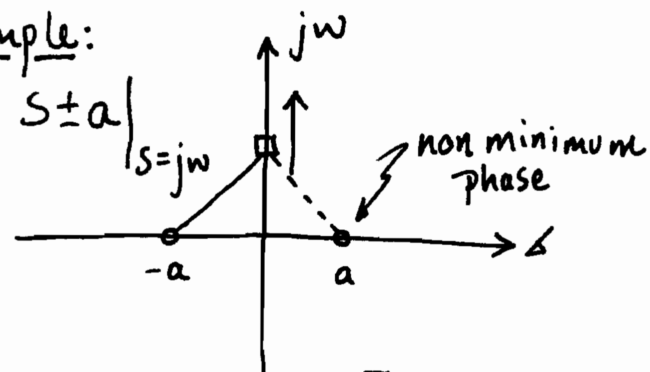
$s_o \to p_1$? $|G(s_o)| \to \infty$

Frequency Response:

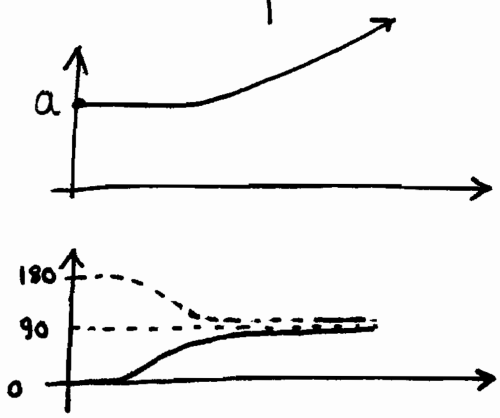


example:

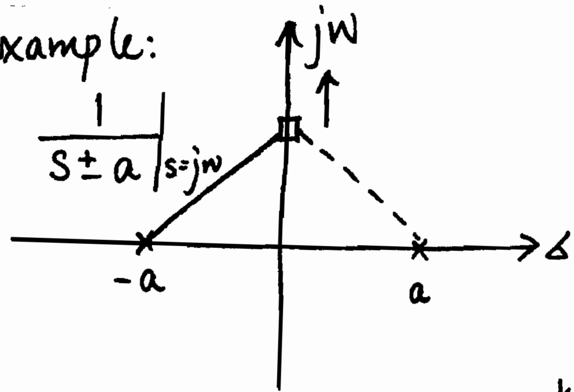
$s \pm a\big|_{s=j\omega}$

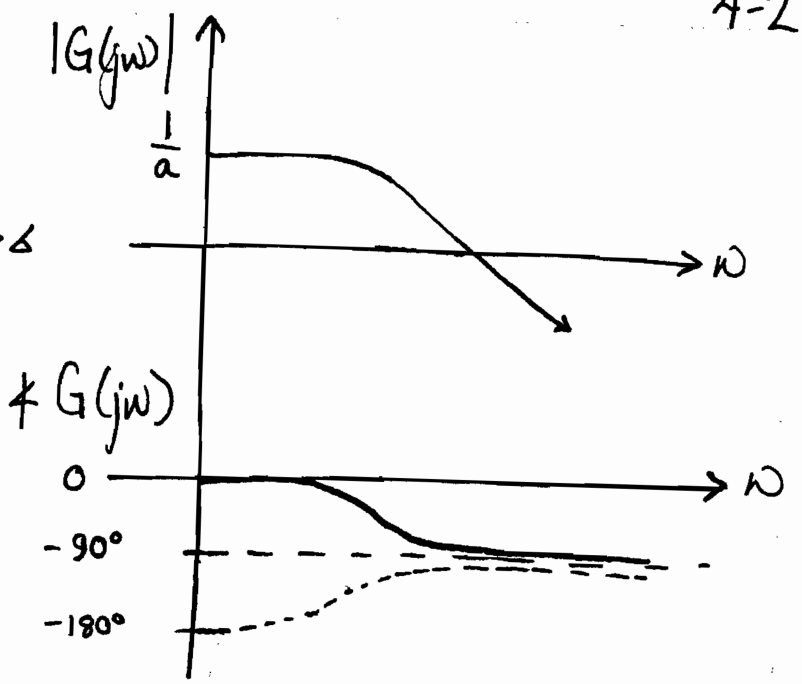


$|G(j\omega)|$
$\measuredangle G(j\omega)$
} Bode plot

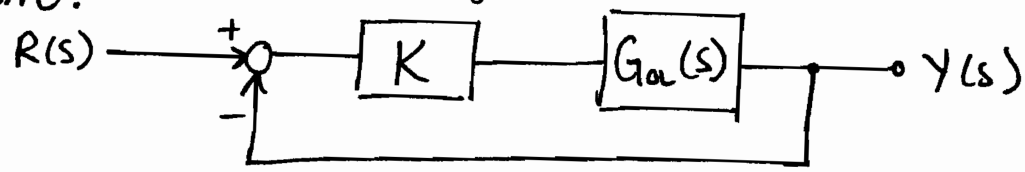


example:





Now, consider the unity feedback closed loop system:



Recall that for $s = j\omega$ to be a closed loop pole, we need that

$$|K G_{OL}(s)| = 1 \quad \text{and} \quad \measuredangle G_{OL}(s) = \pm 180°$$

Thus, let us study

$|K G_{OL}(j\omega)|$ and $\measuredangle G_{OL}(j\omega)$ over a range of ω, and if we find an ω^* at which $|K G_{OL}(j\omega^*)| = 1$ and $\measuredangle G_{OL}(j\omega^*) = \pm 180°$, we recognize that the <u>closed loop</u> system has a pole on the $j\omega$-axis (at $j\omega^*$) and thus the closed loop system is said to be <u>critically</u> (or <u>neutrally</u>) Stable.

BODE PLOT TECHNIQUES:

1. Put the TF into Bode form:

$$K G_{OL}(j\omega) = K \frac{(1+j\omega\tau_1)(1+j\omega\tau_2)\cdots}{(1+j\omega\tau_a)(1+j\omega\tau_b)\cdots}$$

2. Plot gain $|K G_{OL}(j\omega)|$ on a log scale and phase $\measuredangle G_{OL}(j\omega)$ on a linear Scale. Why?

$$|KG_{OL}(j\omega)| = \frac{|K|\,|1+j\omega\tau_1|\,|1+j\omega\tau_2|\cdots}{|1+j\omega\tau_a|\,|1+j\omega\tau_b|\cdots}$$

$$\therefore \log_{10}|KG_{OL}(j\omega)| = \log_{10}|K| + \log_{10}|1+j\omega\tau_1| + \cdots - \log_{10}|1+j\omega\tau_a| - \cdots$$

$\therefore$ on a log scale, we can just <u>add</u> contributions! (in decibels: $|G|_{dB} = 10\log_{10}\frac{P_2}{P_1}$ or $|G|_{dB} = 20\log_{10}\frac{V_2}{V_1}$

Of course, phase is already in additive form:

$$\measuredangle KG_{OL}(j\omega) = \measuredangle K + \measuredangle(1+j\omega\tau_1) + \cdots - \measuredangle(1+j\omega\tau_a) - \cdots$$

So, knowing that we can add magnitudes (and phases) in this way, we can study the contributions from some elementary terms:

<u>example</u>

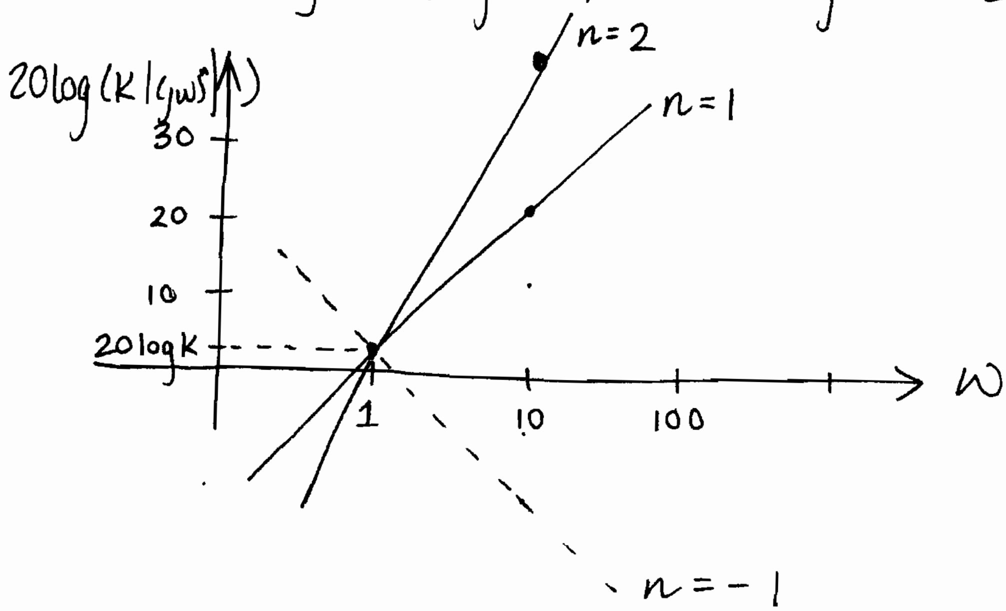
$$KG_{OL}(j\omega) = K\,\frac{(j\omega\tau_1+1)}{(j\omega)^2\,(j\omega\tau_a+1)\left(\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n}+1\right)}$$

Let's look at the Bode plots for these elementary terms:

1. $K(j\omega)^{\pm n}$
2. $(j\omega\tau+1)^{\pm 1}$
3. $\left(\left(\frac{j\omega}{\omega_n}\right)^2 + 2\xi\frac{j\omega}{\omega_n} + 1\right)^{\pm 1}$

1. $K(j\omega)^{\pm n}$:

a. $20\log\left(K\left|(j\omega)^n\right|\right) = 20\log K + 20n\log|j\omega|$



b. $\text{phase}\left(K(j\omega)^n\right) = \text{phase}\ \underbrace{(j\omega)\times(j\omega)\cdots(j\omega)}_{n \text{ times}}$

$= n\cdot 90°$

$= 180°$ for $n=2$
$\quad 90°$ for $n=1$
$\quad -90°$ for $n=-1$.

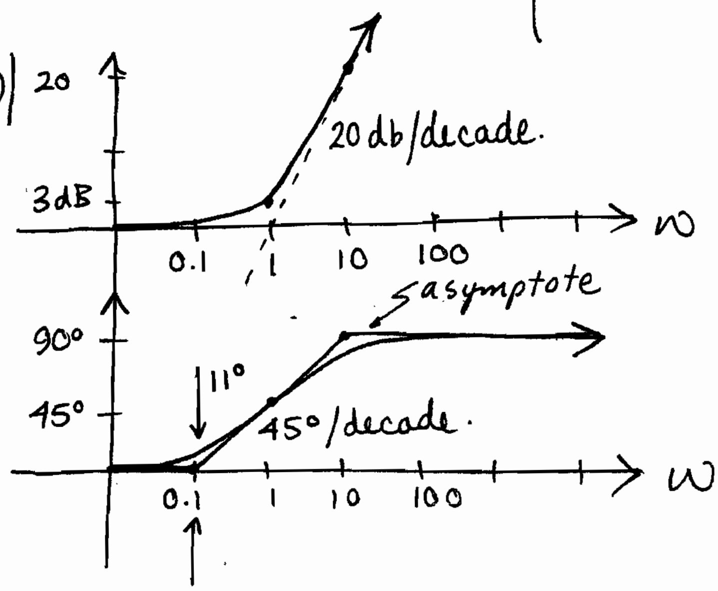
2. $(j\omega\tau+1)^1$

a. $20\log|(j\omega\tau+1)| \doteq \begin{cases} 0 & \omega \to 0 \\ 3\,db & \omega = \frac{1}{\tau} \\ 20\,db & \omega\tau = 10 \\ 40\,db & \omega\tau = 100 \end{cases}$

b. phase $(j\omega\tau+1) \doteq \begin{cases} 0 & \omega \to 0 \\ 45^\circ & \omega = \frac{1}{\tau} \\ 90^\circ & \omega\tau > 10 \end{cases}$

$20\log|(j\omega\tau+1)|$ for $\tau=1$

$\angle(j\omega\tau+1)$

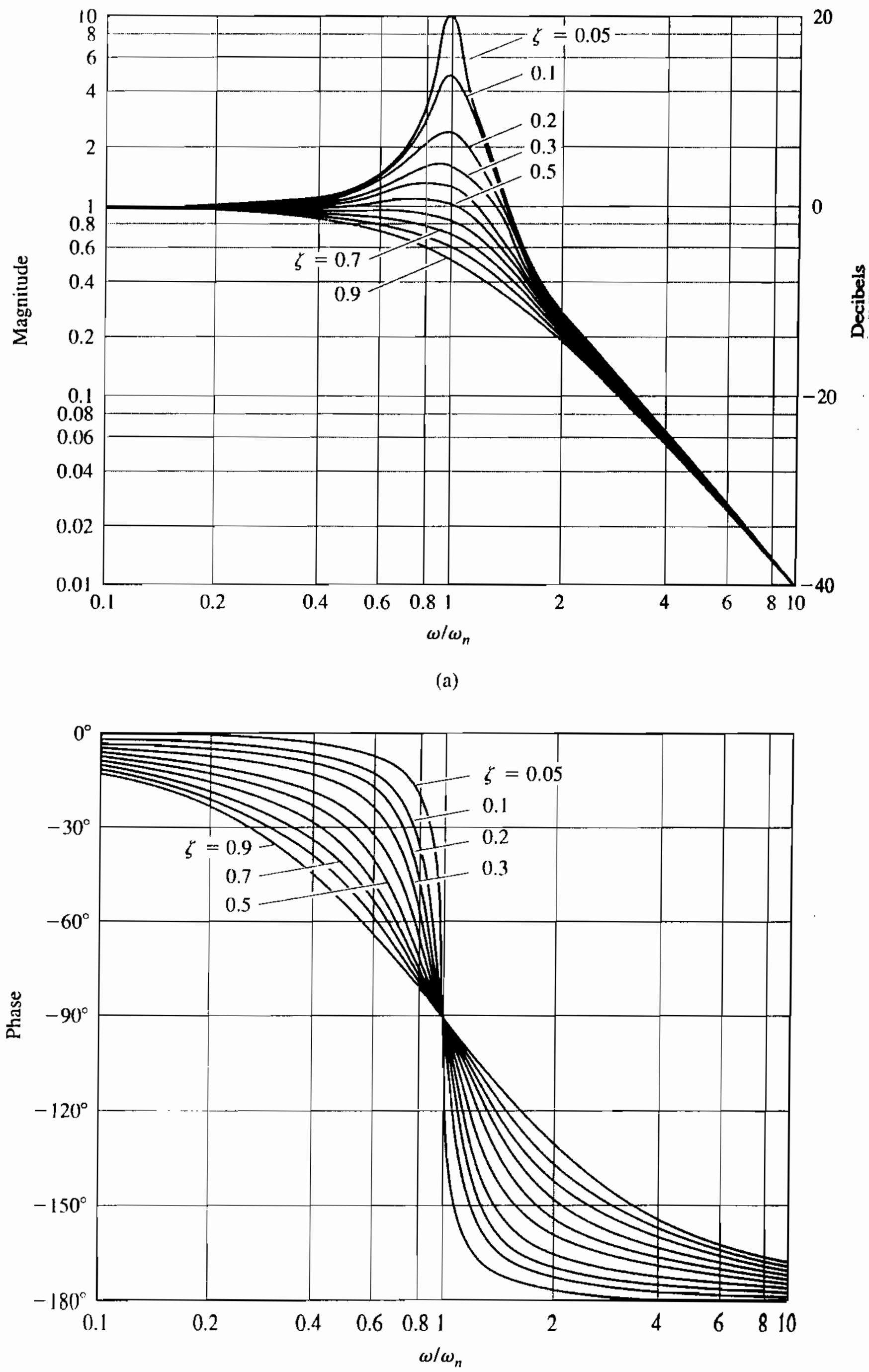


for $(j\omega\tau+1)^{-1}$. just flip above in ω-axis.

3. $\left(\left(\frac{j\omega}{\omega_n}\right)^2 + 2\xi\left(\frac{j\omega}{\omega_n}\right) + 1\right)^{\pm 1}$.

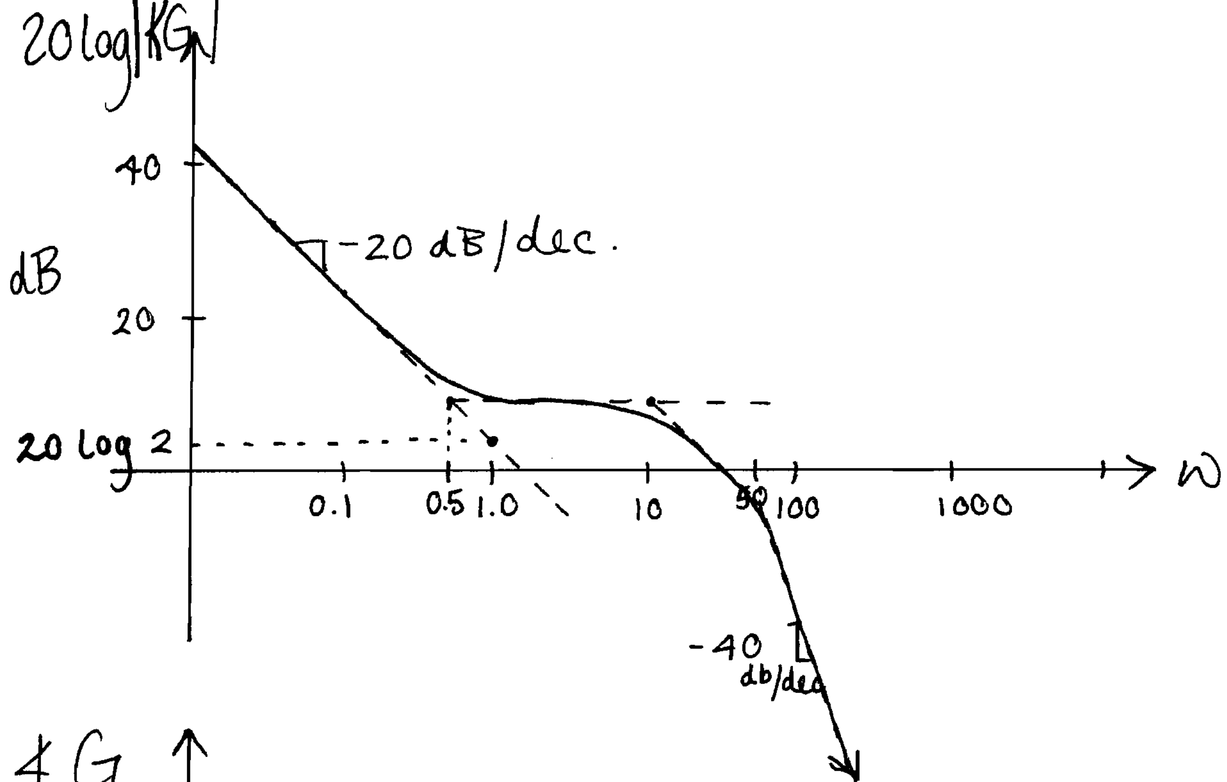
- break point $\omega = \omega_n$
- 40 db/decade. / $\pm 180°$
- ξ affects transition through break point.

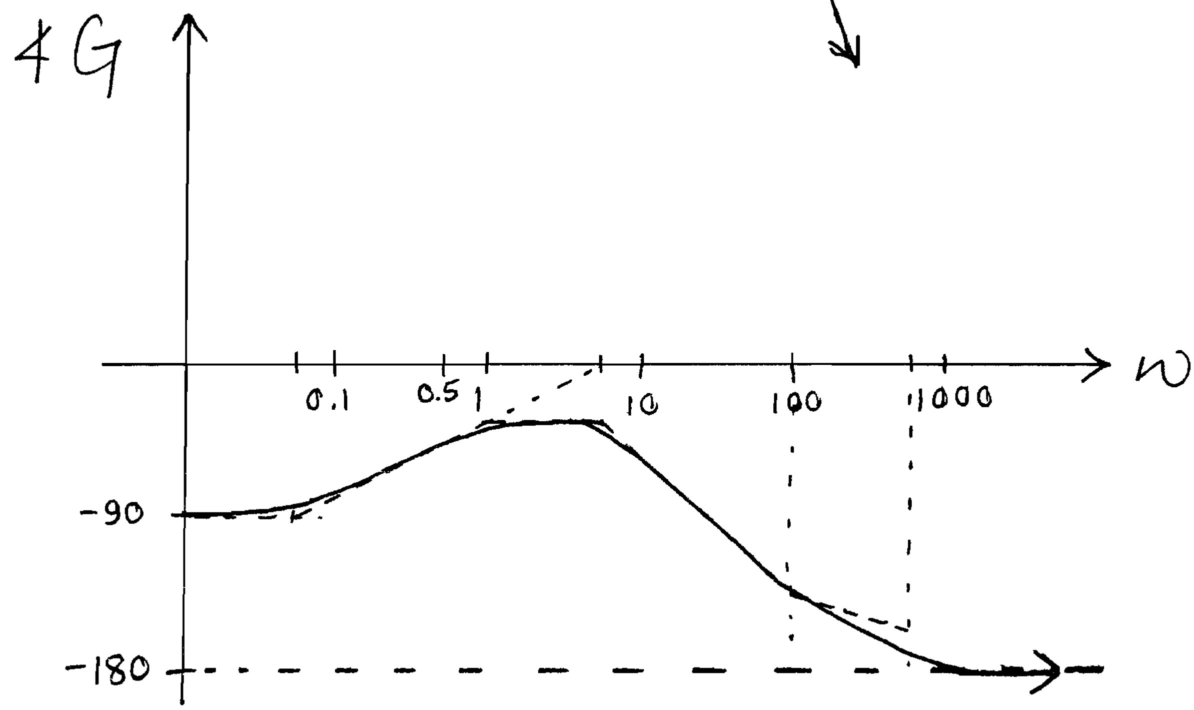
FIGURE 6.2
(a) Magnitude and (b) phase of Eq. (6.7)



EXAMPLE: $KG(s) = \dfrac{2000\,(s+0.5)}{s(s+10)(s+50)}$

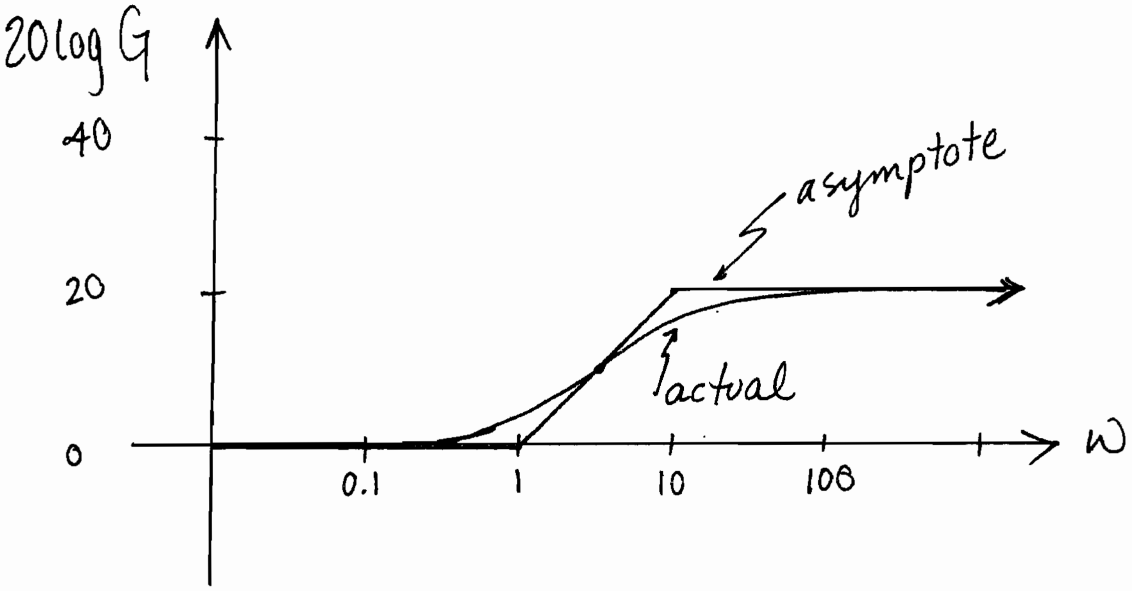
1. $KG(j\omega) = \dfrac{2\left(\frac{j\omega}{0.5}+1\right)}{j\omega\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{50}+1\right)}$

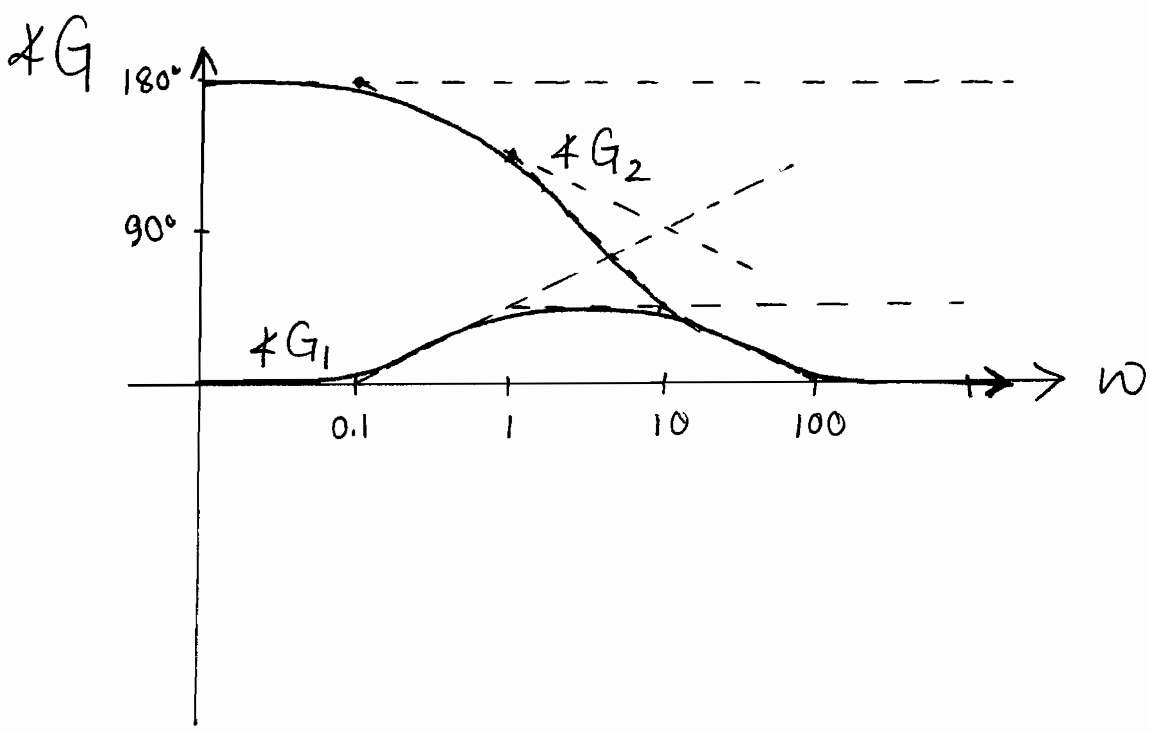




EXAMPLE (non-minimum phase):

$$G_1(s) = \frac{s+1}{\left(\frac{s}{10}+1\right)} \qquad G_2(s) = \frac{s-1}{\left(\frac{s}{10}+1\right)}$$

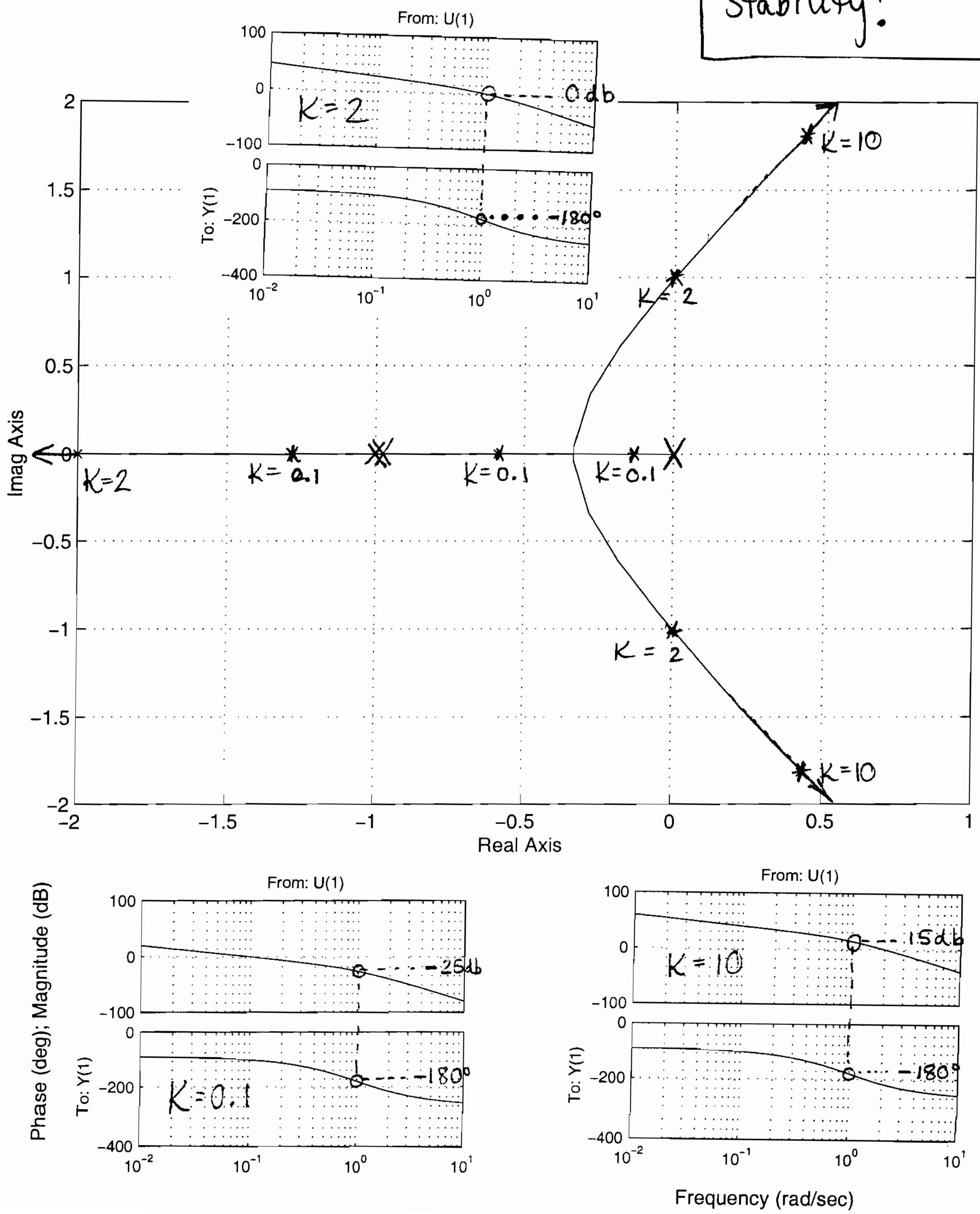




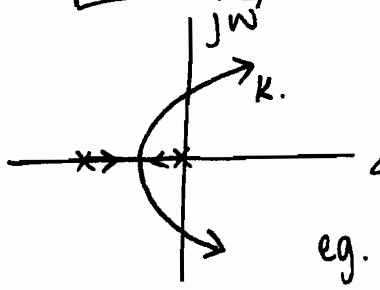
Relating Root Locus to Bode.

$$G(s) = \frac{1}{s(s+1)^2}$$

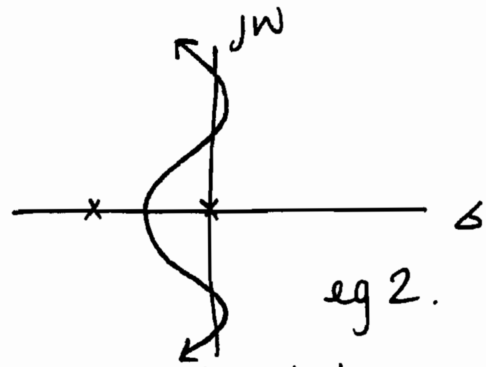
Q: does increasing the gain K increase or decrease the system's stability?



Ans: $\boxed{\text{it depends.}}$



eg. 1



eg 2.

check root locus or Nyquist.

For systems in which increasing gain leads to instability, and $|KG_{OL}(j\omega)|$ crosses the 0 dB line once, the stability condition based on the open loop frequency response is:

$$\boxed{|KG(j\omega)| < 1 \quad \text{at} \quad \measuredangle G(j\omega) = -180^\circ}$$

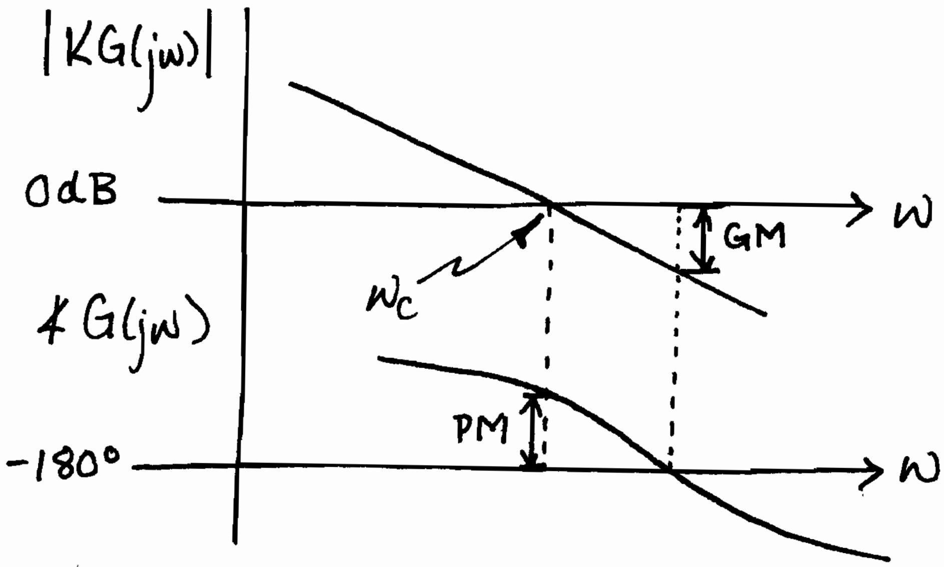
STABILITY CONDITION FOR EG. 1 ABOVE.

For such systems, we define:

Def$^{\underline{n}}$ Gain margin the factor by which K can be increased before instability occurs

Def$^{\underline{n}}$ Phase margin the amount by which the phase of $G(j\omega)$ exceeds -180° when $|KG(j\omega)| = 1$

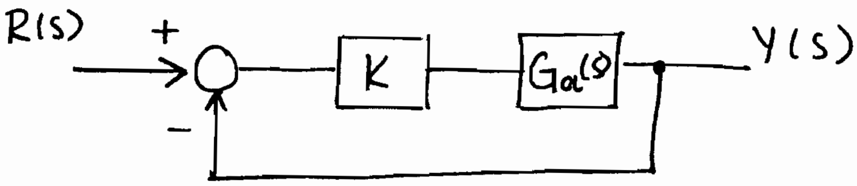
Def^n <u>Crossover frequency</u> is the frequency ω at which $|KG(j\omega)| = 1 = 0\,dB$.



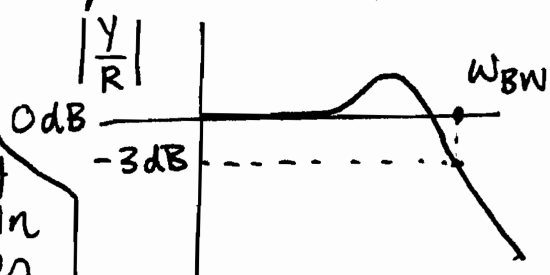
ω_c · crossover frequency

G.M. > 1 for stability
P.M. > 0 for stability

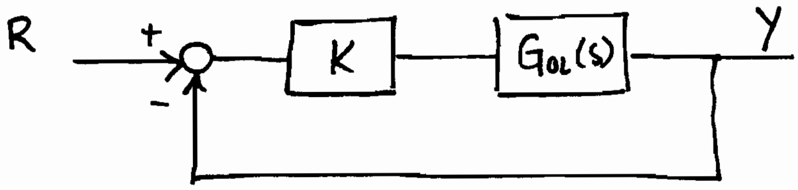
Def^n The <u>bandwidth</u>* ω_{BW} is the frequency of the input at which the output is attenuated to $-3\,dB$.



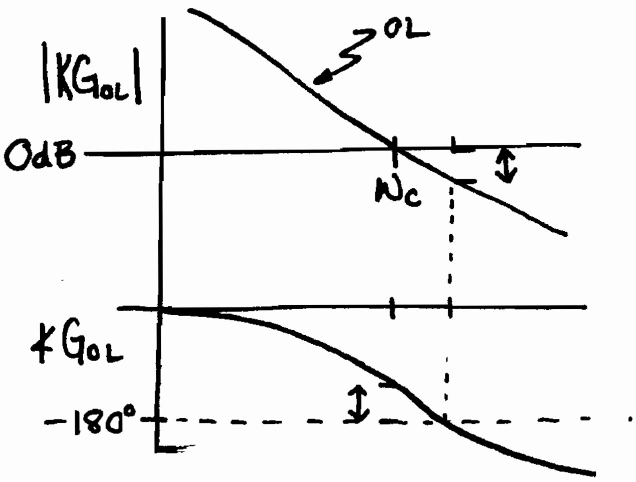
* defined with respect to <u>closed loop</u> <u>bode plot</u>.



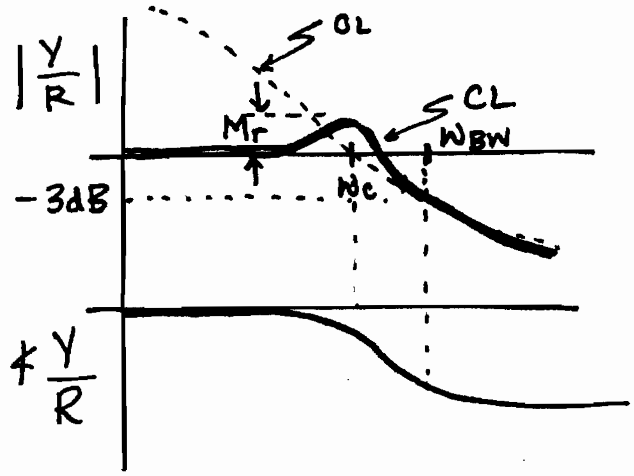
It is the frequency up to which the output will track the input in a satisfactory manner



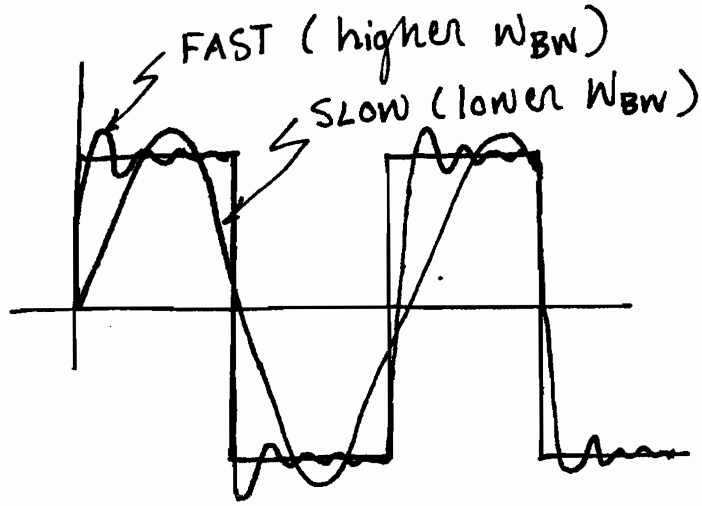
open loop



closed loop.



• BANDWIDTH :



- as $w_{BW} \uparrow$, more components (in Fourier series) pass through. $\Rightarrow$ response fast!
$\Rightarrow$ good tracking.

Q: What is the relationship between $\underbrace{w_c}_{OL}$ and $\underbrace{w_{BW}}_{CL}$?

$$\left|\frac{Y}{R}\right| = \left|\frac{KG(jw)}{1+KG(jw)}\right| \simeq \begin{cases} 1 & w \ll w_c \\ |KG| & w \gg w_c \end{cases}$$

at w_c? $|KG(jw_c)| = 1 \therefore$ phase is important.

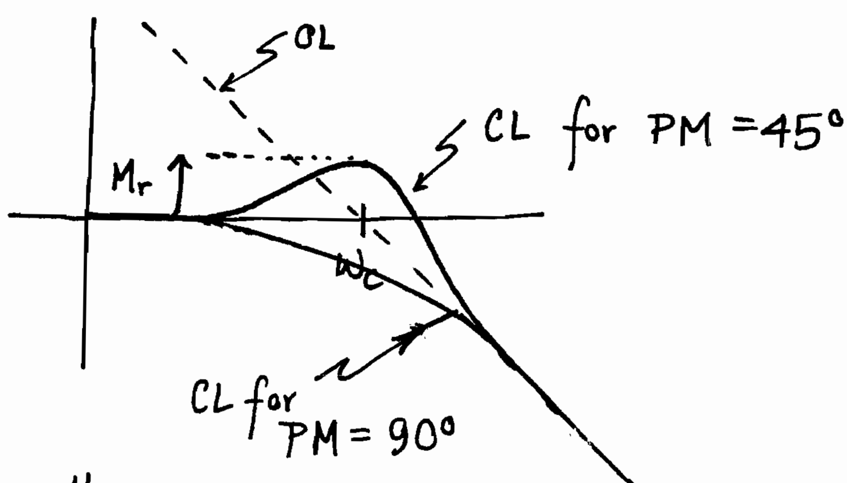
CASE 1 suppose $PM = 90° \Rightarrow \measuredangle KG(jw_c) = -90°$

$$\therefore \frac{Y}{R} = \frac{-j}{1-j} \Rightarrow \left|\frac{Y}{R}\right| = \frac{1}{\sqrt{2}} = 0.707$$

CASE 2 Suppose $PM = 45° \Rightarrow \measuredangle KG(jw_c) = -135°$

$$\therefore \frac{Y}{R} = \frac{1 \cdot \measuredangle -135°}{1 + 1 \cdot \measuredangle -135°}$$

$$\Rightarrow \left|\frac{Y}{R}\right| = 1.31$$

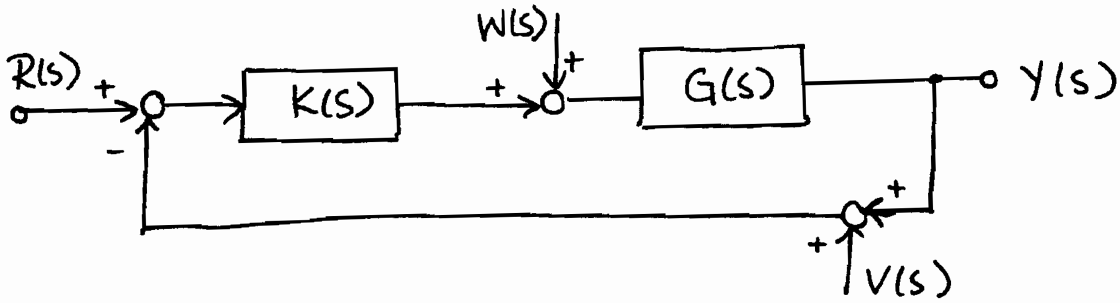


Typically $w_c \leq w_{BW} \leq 2w_c$

2nd order approximations: "Resonant peak": $M_r \simeq \dfrac{1}{2\sin\left(\frac{PM}{2}\right)}$

$\zeta \approx \dfrac{PM}{100}$ for $PM < 70°$

<u>Our GOAL</u>: To design simple compensators $K(s)$ so that $K \cdot G_{OL}(s)$ $[= K(s)\,G(s)]$ has the correct <u>open loop parameters</u> in ω_{BW}, PM, Steady State error, GM to achieve a <u>desired</u> <u>closed loop performance</u>.



$$Y(s) = \frac{G(s)}{1+G(s)K(s)} W(s) + \frac{G(s)K(s)}{1+G(s)K(s)} R(s) \cdots$$
$$- \frac{G(s)K(s)}{1+G(s)K(s)} V(s)$$

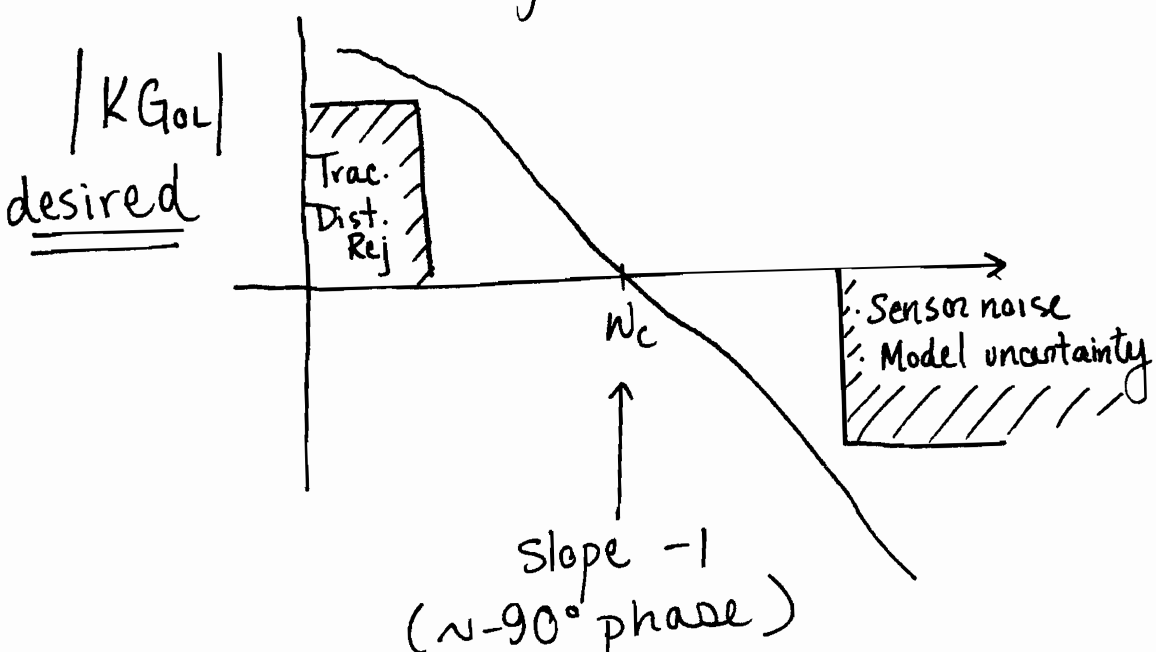
- tracking
- disturbance rejection

} KG ↗ big

- sensor noise
- model uncertainty (high freq.)

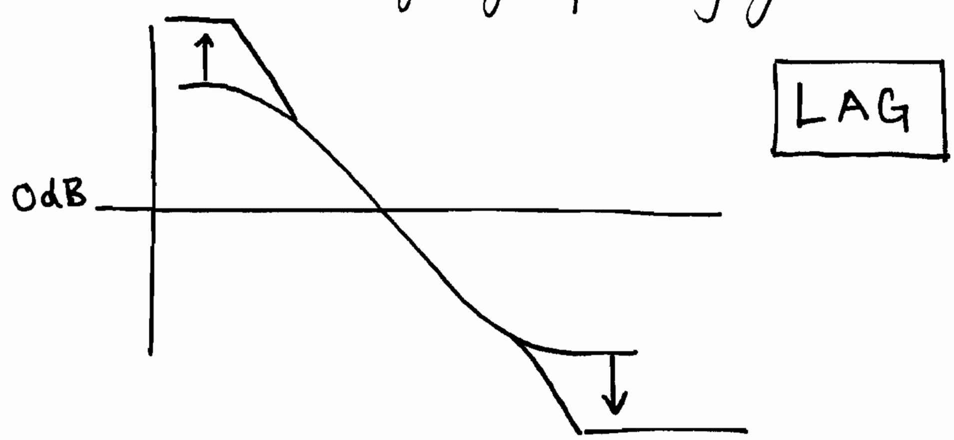
} KG ↘ small

Fortunately, not same frequency range.

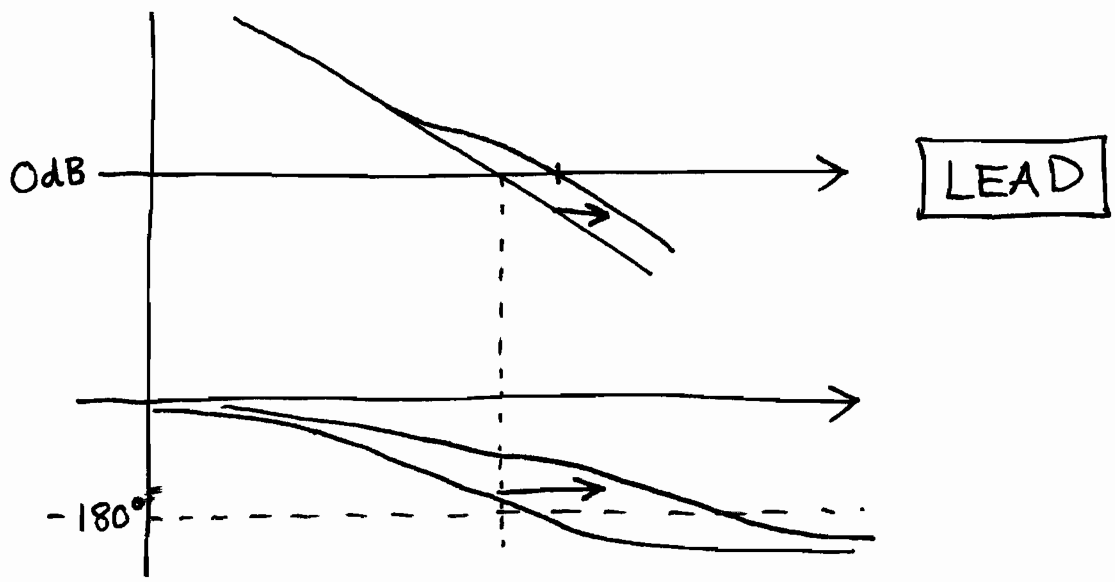


∴ Design tools needed:

(1)
- Raise low frequency gain
- lower high frequency gain

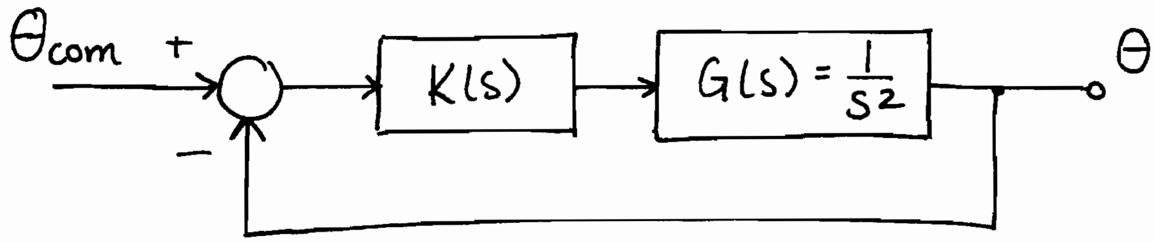


(2)
- GM, PM (for stability, damping)
- BW (transient response)

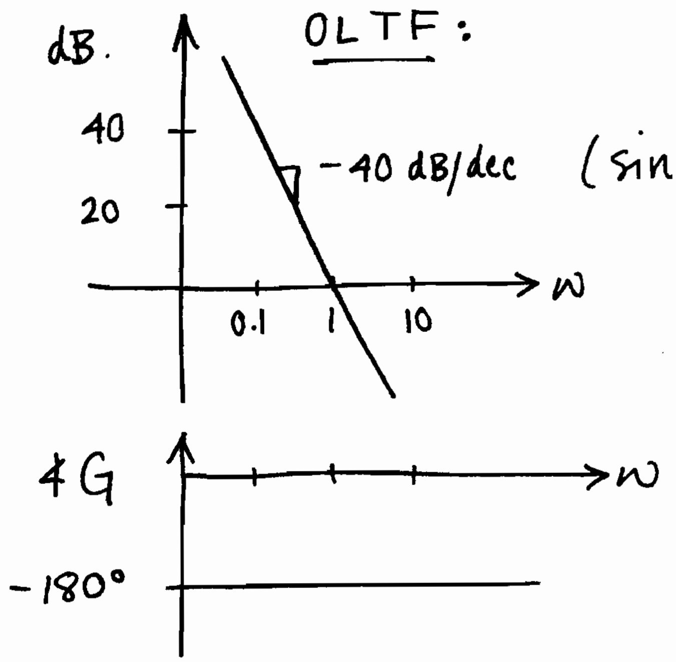


... next Lead-lag Design + examples.

Design example: Spacecraft Attitude Control



Determine K(s) to provide good damping and a bandwidth of approx. 0.2 rad/s.



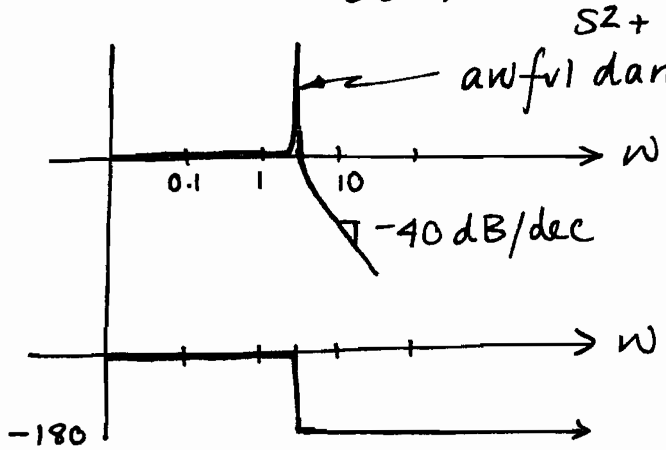
(since $20 \log |(jw)^{-2}| = -40 \log |jw|$

$GM = 1$

$P.M. = 0$

Soln 1 what if $K(s) = K$ ← pure gain?

$\therefore \ CLTF = \dfrac{K}{s^2 + K}$

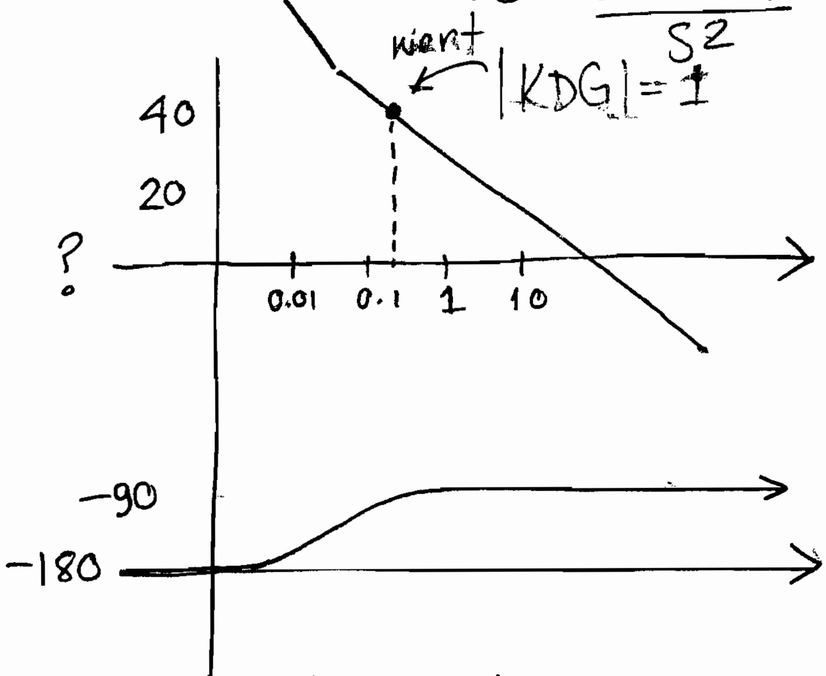


⇐ Bode plot of CLTF

X bad solution

Soln 2 Try $K(s) = K(\tau s + 1)$ (PD)

$\therefore$ OLTF $= K \cdot \dfrac{(\tau s+1)}{s^2}$ let $\tau = \dfrac{1}{w_1}$



$20\log|K| - 40\log|jw| + 20\log|jw\tau+1|$

1. plot $|G(jw)|$
2. decide to use $K(s) = K\overbrace{(\tau s+1)}^{D(s)}$ to improve damping (need to choose K, τ)
3. Spec. BW = 0.2 rad/s
   $\Rightarrow$ $w_c \approx 0.2$ rad/s
   slope = -20 dB/dec @ w_c
   $\therefore$ Choose $w_1 = \frac{1}{\tau} = 0.05$ $(\frac{1}{4} w_c)$
   Choose $|KDG| = 1$ @ w_c
   $\Rightarrow K = \dfrac{1}{|DG|}\Big|_{w=0.2} = \dfrac{1}{10^{\frac{40}{20}}} = 0.01$
4. Validate on CL System: ... over

DESIGN ON OLTF.

Bode Plot of Closed Loop Transfer Function

$$G(s) = \frac{1}{s^2}\,; \quad K(s) = 0.01(20s+1)$$