Due at 1700, Fri. Nov. 13 in homework box under stairs, first floor Cory .
Note: up to 2 students may turn in a single writeup. Reading Nise 5, 12

1. (23 pts) Diagonalization (Nise 5.8)
Consider the linear system:

$$\dot{\mathbf{x}} = A\mathbf{x} + Br = \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} -18 & -1 & -8 \\ 2 & -21 & 2 \\ -6 & 3 & -16 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix} r \quad \text{and} \quad y = [-1\ 1\ 2] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

[14pts] a) Find a diagonal A_z and B_z, C_z such that $\dot{\mathbf{z}} = A_z\mathbf{z} + B_z r$ and $y = C_z\mathbf{z}$. (Matlab use suggested).
[6pts] b) Let $r(t)$ be a unit step. Using Matlab, plot individual states $x_1(t), x_2(t), x_3(t), z_1(t), z_2(t), z_3(t)$.
[3 pts] c. Do the diagonalized states $z_i(t)$ behave the same as the original states $x_i(t)$? Briefly explain why or why not.

2. (20 pts) Control Form transformation (Nise 12.4)
Given the following: $\dot{\mathbf{z}} = A\mathbf{z} + Bu = \begin{bmatrix} -21 & 5 \\ -15 & 1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} u(t), \quad y = [1\ 0]\ \mathbf{z}$
[10pts] a) Find the transformation P such that $(\bar{A}, \bar{B})$ is in phase variable form, where $\bar{A} = P^{-1}AP$ and $\bar{B} = P^{-1}B$.
[10pts] b) Find $\bar{A}, \bar{B}, \bar{C}$ such that $\dot{\bar{x}} = \bar{A}\bar{\mathbf{x}} + \bar{B}u$ and $y = \bar{C}\bar{x}$.

3. (10 pts) Controllability (Nise 12.3)

Consider the system $\dot{\mathbf{x}} = \begin{bmatrix} k_1 & -2 \\ -1 & k_2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u(t)$. For what values of k_1 and k_2 is the system completely controllable?

4. (27 pts) Observer (Nise 12.5)
Given the plant:

$$G(s) = \frac{10}{(s+3)(s+7)(s+10)}$$

where state variables are not available.
[6pts] a. Express $G(s)$ in observer canonical form, $\dot{x} = Ax + Bu$.
[15pts] b. Design an observer: $\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y})$ for the observer canonical variables to yield a 2nd order transient response with $\zeta = 0.5$ and $\omega_n = 50$. (The third pole should be placed 10 times further from the imaginary axis than the dominant poles.)
[6pts] c. Using Matlab, compare the state variables in G for a step input with the observer estimate. (That is, plot $\mathbf{x}(t)$ and $\hat{\mathbf{x}}(t)$.)

5. (20 pts) Controllability and Observability (Nise 12.3, 12.6)
For the circuit below, input is voltage $v_i(t)$, output is voltage $v_c(t)$, states x_1 = capacitor voltage, and x_2 = inductor current.
[5pts] a) Write state and output equations for the circuit.
[5pts] b) Find conditions for R_2, C that make the system controllable and observable.
[5pts] c) Interpret the conditions that make the system lose controllability or observability in terms of the time constants of the system.