1. (10 pts) Case study (Nise 1.4)
In a nuclear power plant, the rate of the fission reaction determines the amount of heat generated, and this rate is controlled by rods inserted into the radioactive core. The rods regulate the generation of neutrons. Inserting rods decreases rate of fission and removing rods increases fission rate. Draw a functional block diagram for the nuclear reactor control system shown in Fig. 1. Show all blocks and signals.

2. (15 pts) Static Nonlinearity in Feedback
A nonlinear amplifier has voltage response \( g(\epsilon) = 1000\epsilon \) for \( 0 \leq \epsilon < 0.5 \) and \( g(\epsilon) = 2000\epsilon \) for \( 0.5 \leq \epsilon \leq 1 \). The nonlinear amplifier is used in a negative feedback system as shown in Fig. 2, with \( k = 0.01 \). Let \( x(t) = 1 + \cos \omega_0 t \).

(a) Let \( \delta = 0 \). Show that \( y(t) \approx 100x(t) \). (Hint: assume \( \epsilon \) is small and check that the assumption holds.) How close is the closed loop amplifier to a gain of 100 for the given input signal?

(b) Let \( \delta = 0.1 \). What effect does this offset have on the output \( y(t) \)?

3. (15 pts) Laplace transform review (Nise 2.2)
For each transfer function below determine \( h_i(t) \).

\[
\begin{align*}
   i) H_1(s) &= \frac{1}{s^2+22s+40} & \text{ii) } H_2(s) &= \frac{s}{s^2+22s+40} & \text{iii) } H_3(s) &= \frac{s+10}{s^2+22s+40} \\
   iv) H_4(s) &= \frac{1}{s^2+20s+101} & \text{v) } H_5(s) &= \frac{1}{s^3+22s^2+40s}
\end{align*}
\]

4. (15 pts) Initial value, final value (Nise 2.2)
For each of the following Laplace transforms \( Y_i(s) \) determine \( y_i(t=0^+) \) and if the limit exists, \( \lim_{t \to \infty} y_i(t) \):

\[
\begin{align*}
   i) Y_1(s) &= \frac{s}{s+4} & \text{ii) } Y_2(s) &= \frac{s+3}{s+4} & \text{iii) } Y_3(s) &= \frac{s-3}{s(s+4)} \\
   iv) Y_4(s) &= \frac{1}{s(s+4)} & \text{v) } Y_5(s) &= \frac{(s+3)^2}{s^2}
\end{align*}
\]

5. (15 pts) Electrical circuit example (Nise 2.4)
For the circuit in Fig. 3. below, using ideal op-amp assumptions, determine \( H(s) = \frac{V_{\text{output}}(s)}{V_{\text{input}}(s)} \).

6. (30 pts) Equivalent electrical and mechanical circuits (Nise 2.8, 2.9)
Consider the model of a piezoelectric element (used for example in energy harvesting). Assume the current generated by the piezo element is proportional to velocity \( I_{\text{piezo}} = \alpha \dot{x} \) and the generated force is proportional to voltage \( F_{SL} = \beta V_{\text{piezo}} \).

(a) Write the transfer function for the electrical input impedance \( \frac{V_{\text{piezo}}}{I_{\text{piezo}}} \).

(b) Draw the equivalent electrical circuit for the system in Fig. 4, (with voltage corresponding to force, and current corresponding to velocity).

(c) Draw the equivalent mechanical circuit for the system in Fig. 4, (with voltage corresponding to force, and current corresponding to velocity).