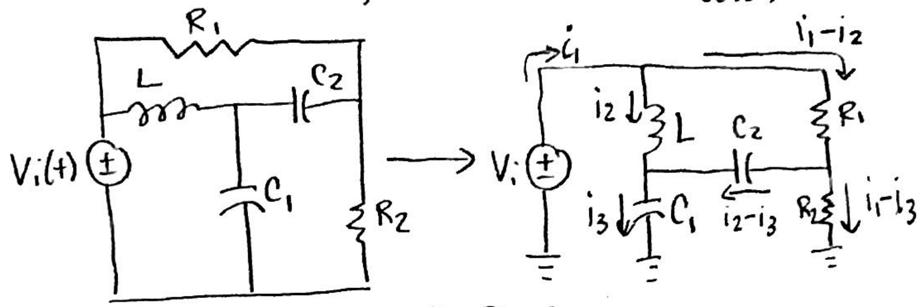
1 There are several ways to solve this ... we present two methods.

(a) (i) Redraw the circuit and redefine new currents so the inductor analog (mass) is not between two diff. positions

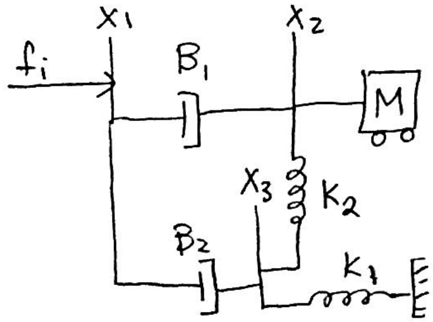


* currents through C_2, R_1, R_2 obtained using nodal analysis.

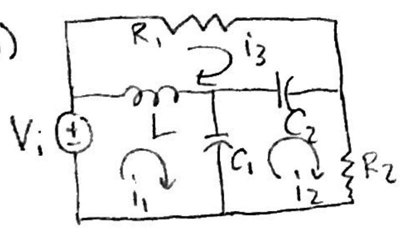
(ii) Draw analog elements according to relative positions. For example, R_1 has current $i_1 - i_2$, so B_1 must sit between x_1 and x_2 b/c its velocity is $v_1 - v_2$.

Let $f_i = V$
$M = L$
$K_1 = 1/C_1$
$K_2 = 1/C_2$
$R_1 = B_1$
$R_2 = B_2$

Then



(b)



using method described @: lpsa.swarthmore.edu/Analogs/RelElecMech2.html

(i) Write the loop equations

$$\sum_{\text{loop } i_1} V: 0 = V_i + L\frac{d(i_3 - i_1)}{dt} + \frac{1}{C_1}\int (i_2 - i_1)\,dt$$

$$\sum_{\text{loop } i_2} V = 0 = \frac{1}{C_1}\int (i_2 - i_1)\,dt + \frac{1}{C_2}\int (i_2 - i_3)\,dt + i_2 R_2$$

$$\sum_{\text{loop } i_3} V = 0 = i_3 R_1 + \frac{1}{C_2}\int (i_3 - i_2)\,dt + L\frac{d(i_3 - i_1)}{dt}$$

(ii) Write analogous quantities

$$\sum_{x_1} f = 0 = f_i + M(\ddot{x}_3 - \ddot{x}_1) + k_1(x_2 - x_1)$$

$$\sum_{x_2} f = 0 = k_1(x_2 - x_1) + k_2(x_2 - x_3) + B_2\dot{x}_2$$

$$\sum_{x_3} f = 0 = B_1\dot{x}_3 + k_2(x_3 - x_2) + M(\ddot{x}_3 - \ddot{x}_1)$$

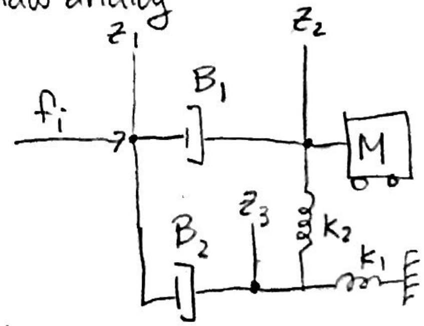
(iii) Redefine positions (choose positions to get rid of difference of positions for mass and simplify some expressions)

Let $z_1 = -x_1$
$z_2 = x_3 - x_1$
$z_3 = x_2 - x_1$

(iv) Rewrite equations

$$0 = f_i + M\ddot{z}_2 + k_1 z_3$$
$$0 = k_1 z_3 + k_2(z_3 - z_2) + B_2(\dot{z}_3 - \dot{z}_1)$$
$$0 = B_1(\dot{z}_2 - \dot{z}_1) + k_2(z_2 - z_3) + M\ddot{z}_2$$

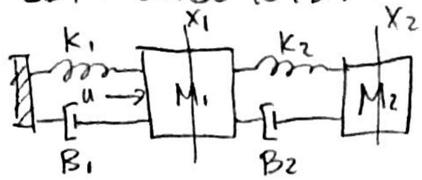
(v) Draw analog

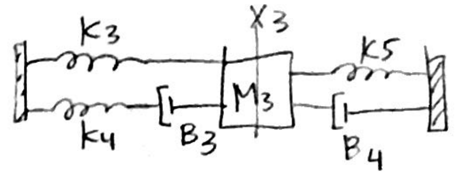


(vi) ... which is equivalent to that of part (a). Notice that the 2nd and 3rd equations give you everything in the mechanical circuit except for force, and the 1st equation connects all the "appendages" to the loop corresponding to the original node @ V_i.

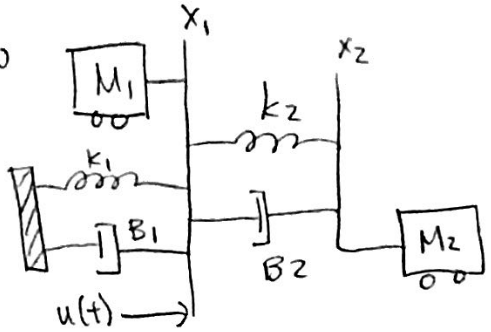
$\boxed{2}$ (a) LEFT SUBSYSTEM RIGHT SUBSYSTEM

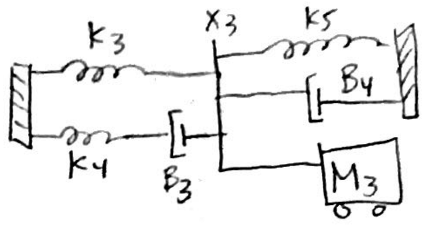
(i) original



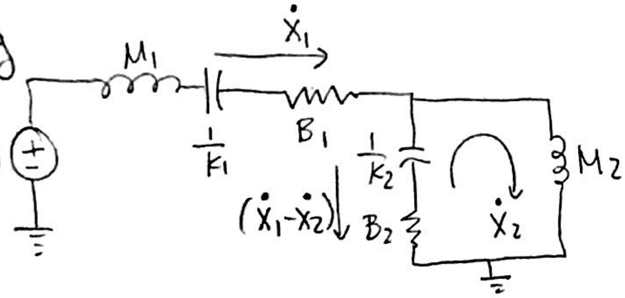


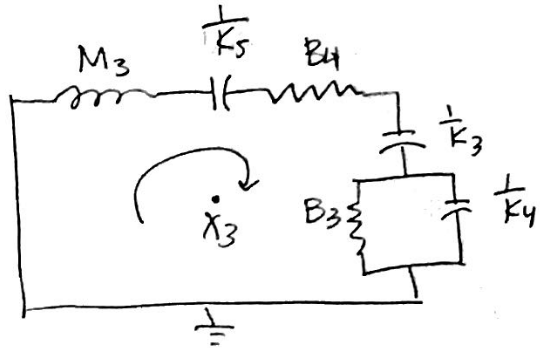
(ii) redraw





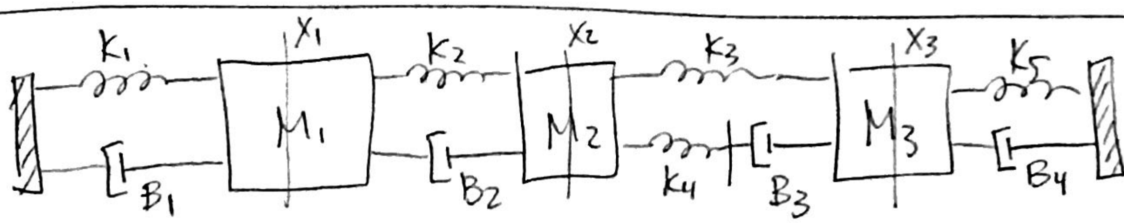
(iii) analog



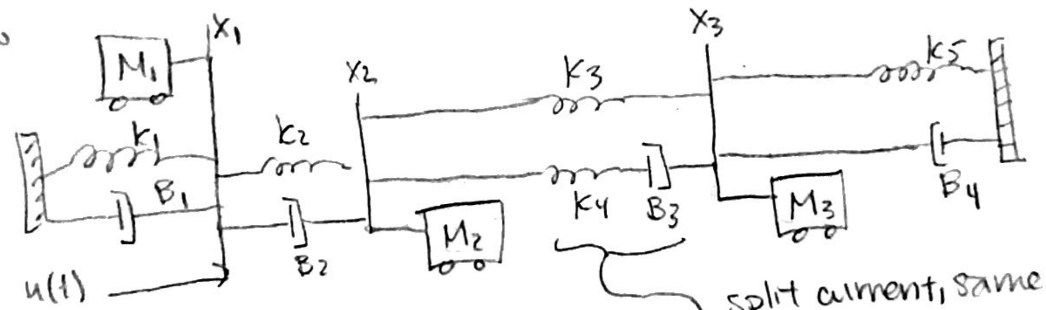


(b)

(i) original

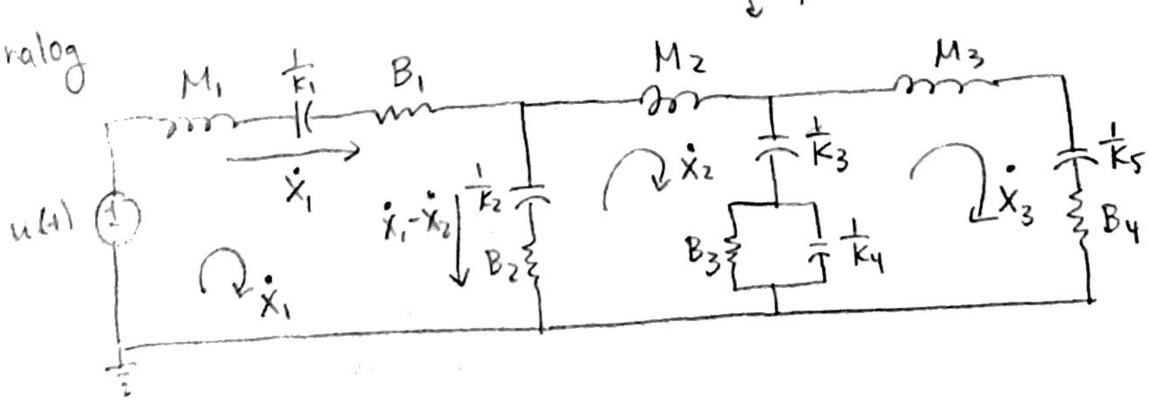


(ii) redraw



split current, same force on branch → parallel

(iii) analog



3 Linearization

$x(t), f() \rightarrow \ddot{x}(t)+16\dot{x}(t)+55x=f(x,t)$, $f(x,t)=e^{1-x}$ for $t\geq 0$, $f(x,t)=0$ for $t<0$

(a) Linearize near $x\approx 1$

Let $x=1+\delta x$

$$\frac{d^2(\delta x+1)}{dt^2}+16\frac{d(\delta x+1)}{dt}+55(\delta x+1)-e^{1-(1+\delta x)}=0$$

but $\frac{d^2(\delta x+1)}{dt^2}=\frac{d^2(\delta x)}{dt^2}$ and $\frac{d(\delta x+1)}{dt}=\frac{d(\delta x)}{dt}$ and $e^{1-(1+\delta x)}=e^{-\delta x}$

$$\rightarrow \frac{d^2\delta x}{dt^2}+16\frac{d\delta x}{dt}+55\delta x+55-e^{-\delta x}=0$$

Linearize $e^{-\delta x}$: Use truncated Taylor series

$$f(\delta x)=e^{-\delta x},\ f'(\delta x)=-e^{-\delta x},\ f(0)=1,\ f'(0)=-1$$

$$L(\delta x)=1-1(\delta x-0) \rightarrow e^{-\delta x}\approx 1-\delta x$$

Then $\frac{d^2\delta x}{dt^2}+16\frac{d\delta x}{dt}+55\delta x+55-1+\delta x=0$

$$\hookrightarrow \boxed{\frac{d^2\delta x}{dt^2}+16\frac{d\delta x}{dt}+56\delta x+54=0} \text{ around } x\approx 1$$

(b) IC: $x(0^-)=1$, $\dot{x}(0^-)=0.1$, Find $X(s)$

Plug in $x-1=\delta x$ into linearized form from part (a):

$$\frac{d^2(x-1)}{dt^2}+16\frac{d(x-1)}{dt}+56(x-1)+54=0 \rightarrow \frac{d^2x}{dt^2}+16\frac{dx}{dt}+56x-2=0$$

We know that $\mathcal{L}\{\dot{x}\}=sX(s)-x(0)$ and $\mathcal{L}\{\ddot{x}\}=s^2X(s)-sx(0)-\dot{x}(0)$

Then $\mathcal{L}\{\frac{d^2x}{dt^2}+16\frac{dx}{dt}+56x-2\}=0$

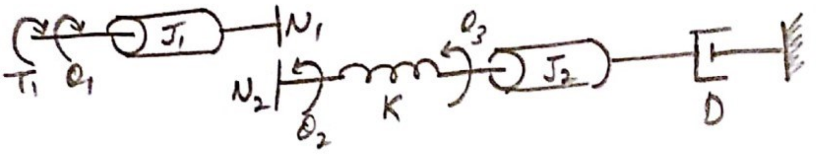
$$\rightarrow s^2X(s)-sx(0)-\dot{x}(0)+16(sX(s)-x(0))+56X(s)-2\frac{1}{s}=0$$

$$\rightarrow s^2X(s)-s-0.1+16sX(s)-16+56X(s)-\frac{2}{s}=0$$

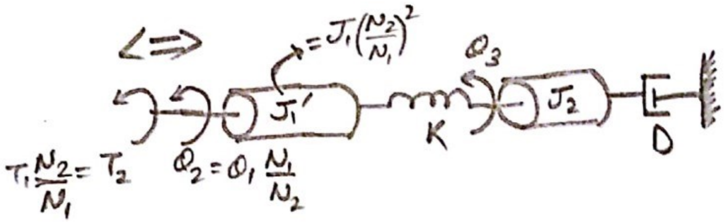
$$\rightarrow X(s)(s^2+16s+56)=s+\frac{2}{s}+16.1$$

$$\rightarrow \boxed{X(s)=\frac{s+\frac{2}{s}+16.1}{s^2+16s+56}}$$

HW2 P4



$\Longleftrightarrow$



(a)

eq^n ①: $\quad T_2 = J_1' s^2 \theta_2 + K(\theta_2 - \theta_3)$

eq^n ②: $\quad 0 = J_2 s^2 \theta_3 + Ds\,\theta_3 + K\theta_3 - K\theta_2$

$$\Rightarrow \theta_3 = \frac{K}{J_2 s^2 + Ds + K}\,\theta_2$$

Substitute in eq^n ①:
$$T_2 = J_1' s^2 \theta_2 + K\theta_2\left(1 - \frac{K}{J_2 s^2 + Ds + K}\right)$$

$$T_1\frac{N_2}{N_1} = \left[J_1\left(\frac{N_2}{N_1}\right)^2 s^2 \theta_1 + K\theta_1\left(\frac{J_2 s^2 + Ds}{J_2 s^2 + Ds + K}\right)\right]\frac{N_1}{N_2}$$

$$\Rightarrow \frac{T_1}{\theta_1} = \left(J_1\left(\frac{N_2}{N_1}\right)^2 s^2 + \frac{(J_2 s^2 + Ds)K}{J_2 s^2 + Ds + K}\right)\left(\frac{N_1}{N_2}\right)^2$$

$J_1 = 50\ \text{kgm}^2$
$J_2 = 100\ \text{kgm}^2$
$K = 100\ \frac{\text{Nm}}{\text{rad}}$
$D = 100\ \frac{\text{Nms}}{\text{rad}}$
$N_1 = 30,\ N_2 = 100$

$$= \left(50\left(\frac{100}{9}\right)s^2 + 100\,\frac{(100s^2 + 100s)}{100s^2 + 100s + 100}\right)\left(\frac{9}{100}\right)$$

$$= \left(50s^2 + \frac{9s^2 + 9s}{s^2 + s + 1}\right) = \frac{50s^4 + 50s^3 + 59s^2 + 9s}{s^2 + s + 1}$$

$$\Rightarrow \boxed{\frac{\theta_1(s)}{T_1(s)} = \frac{s^2 + s + 1}{50s^4 + 50s^3 + 59s^2 + 9s}}$$

(b) $$\frac{Q_1(s)}{T_1(s)} = \frac{\frac{1}{50}(s^2+s+1)}{s^4+s^3+\frac{59}{50}s^2+\frac{9}{50}s}$$

$$\Rightarrow \quad Q_1(s) = \frac{1}{50}(s^2+s+1)\,X(s)$$ for some intermediate function $X(s)$

and, $$X(s) = \frac{T_1(s)}{s^4+s^3+\frac{59}{50}s^2+\frac{9}{50}s}$$

$$\Rightarrow \quad \frac{d^4x}{dt^4} = t_1 - \frac{d^3x}{dt^3} - \frac{59}{50}\frac{d^2x}{dt^2} - \frac{9}{50}\frac{dx}{dt}$$

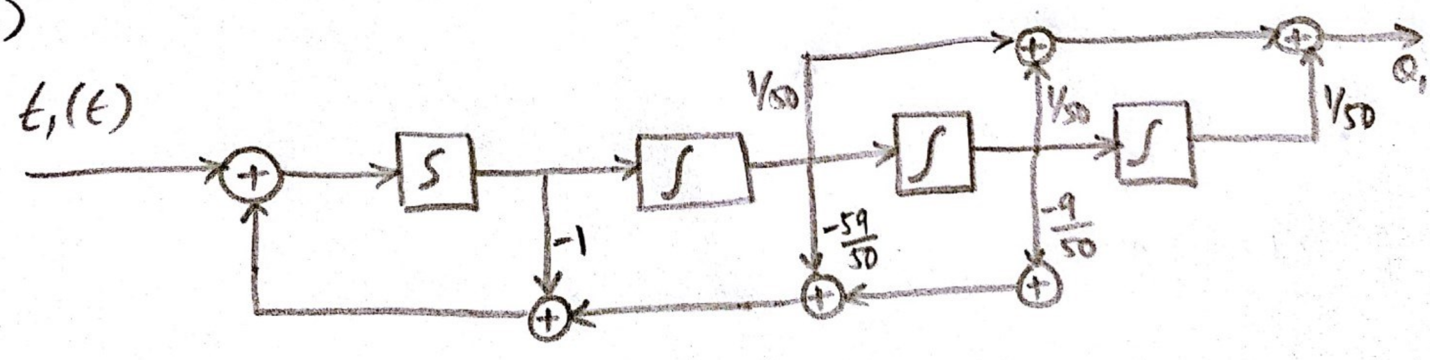
for $x_1 = x$, $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = x_4$ and $\dot{x}_4 = \frac{d^4x}{dt^4}$

$$\dot{\underline{x}} = A\underline{x} + Bt_1 = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -9/50 & -59/50 & -1\end{bmatrix}}_{\boxed{A}}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix}}_{\boxed{B}} t_1$$

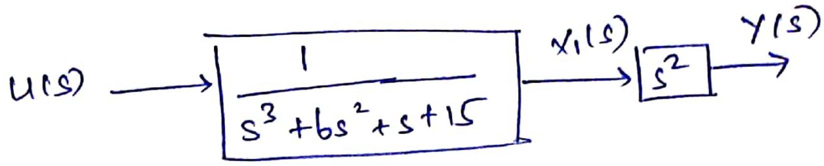
Also, $$Q_1(t) = \frac{1}{50}\frac{d^2x}{dt^2} + \frac{1}{50}\frac{dx}{dt} + \frac{1}{50}x$$

$$Q_1 = C\underline{x} + Dt_1 = \underbrace{\begin{bmatrix}1/50 & 1/50 & 1/50 & 0\end{bmatrix}}_{\boxed{C}}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \underbrace{0}_{\boxed{D}}\, t_1$$

(c)



Q5. $\dfrac{Y(s)}{U(s)} = \dfrac{s^2}{s^3 + 6s^2 + s + 15}$



$$U(s) \dfrac{1}{s^3 + 6s^2 + s + 15} = X_1(s)$$

$$U(s) = X_1(s)\left[s^3 + 6s^2 + s + 15\right]$$

inverse Laplace transforms :-

$$u = \dddot{x}_1 + 6\ddot{x}_1 + \dot{x}_1 + 15x_1$$

let $x_2 = \dot{x}_1$

$x_3 = \dot{x}_2$

$\dot{x}_3 = \dot{x}_3 = u - 15x_1 - x_2 - 6x_3$

$\therefore$ or

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = u - 15x_1 - x_2 - 6x_3$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -1 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

Now, moving to the second block in the diagram.

$$s^2 X_1(s) = Y(s)$$

taking inverse laplace transform

$y = \ddot{x}_1$ since $x_1 = x_1$

$\dot{x}_1 = x_2$

$\ddot{x}_1 = x_3$

$y = \ddot{x}_1 = x_3$ $y = [0 \quad 0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

since a state space equation can be expressed as:-

$\dot{x} = Ax + Bu$

$y = Cx + Du$

therefore $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -15 & -1 & -6 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$C = [0 \quad 0 \quad 1]$ $D = 0$