Professor Fearing EECS128/ME134 Problem Set 3 v1.00 Fall 2018

Due at 1700, Fri. Sep. 14 on bcourses .

Note: up to 2 students may turn in a single writeup. Reading Nise 4. Note: eqn 4.45: $\frac{4}{\sigma_d}$

1. (20 pts) Linearization (Nise 3.7)

For the system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \cos x_2 - \sin x_1 \\ u^2 - x_3^2 \\ \frac{1}{u} + x_4 \\ x_3 - \sqrt{2u} \end{bmatrix} \tag{1}$$

Linearize the system about $x_1 = x_2 = \pi/4, x_3 = 2, x_4 = \frac{-1}{2}$, $u = 2$, and express in state space form:

$\dot{\delta}\mathbf{x} = \mathbf{A}\delta\mathbf{x} + \mathbf{B}\delta u$.

2. (15 pts) First order step response and PWM (Nise 4.3)

Actuators in many control systems are driven by a pulse-width-modulated (PWM) signal as suggested in Fig. 1. The pulse signal is either smoothed by the plant being controlled or a filter can be used, for example in audio applications. Consider a PWM signal $\{0, 10\}$ volts, driving a filter with transfer function $H(s) = \frac{1}{s+1}$.

`[2 pts]` a. What duty cycle (T_{on}/T_{period}) would be used to generate a filter output of an approximately constant 2.0 volts?

`[13 pts]` b. Consider steady-state operation with duty cycle as found in a. What is the maximum T_{period} to have the output voltage in the range 2.0 ± 0.01 volts?

3. (20 pts) 2nd order step response (Nise 4.6)

For the system shown in Fig. 2, find the values of R, L and C such that $v_c(t)$ has a 10% overshoot with a time to peak $T_{peak} = 1ms$ with $v_i(t)$ a step input. Also note ζ and ω_n.

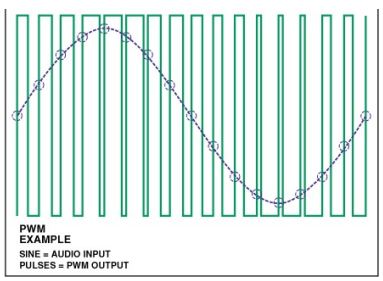

Fig. 1 www.analog.com

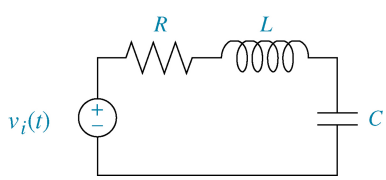

Fig. 2

4. (15 pts) Second order poles (Nise 4.6)

For each pair of second-order system step response specifications, find the location of the second order pair of poles.

a. %OS = 10%; $T_s = 0.1$ seconds.

b. %OS = 80%; $T_s = 0.1$ seconds.

c. %OS = 10%; $T_p = 0.1$ sec.

5. (15 pts) Dominant poles (Nise 4.7)

A system has transfer function $H(s) = \frac{a}{(s+1)(s+a)}$. Consider the step reponse of the system $= y(t)$. The claim is made that for some range of a, $y(t) \approx (1 - e^{-t})u(t)$, i.e. the pole at $s = -a$ can be neglected. If so, find the range of a for which the approximate step response is within 1% of the true step response for $H(s)$. Also provide a plot using Matlab for the true and approximate step response.

6. (15 pts) Time Domain Solution- Laplace (Nise 4.10)

Given the following state-space representation find $y(t)$ using the Laplace transform method (by hand calculation). Here $u(t)$ is the unit step.

$$\dot{\mathbf{x}} = A\mathbf{x} + Bu = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -5 & 10 \\ 2 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t) \ , \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \ y = [4 \quad 1]\mathbf{x}$$