1 $\dot{x} = Ax + Bu = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & -4 & -12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t), \quad y = [0\ 0\ 1]\,x$

$$\det(sI-A) = \det\begin{bmatrix} s+3 & 4 & 12 \\ -1 & s & 0 \\ 0 & -1 & s \end{bmatrix} = (s+3)(s^2) - 4(-s) + 12(1)$$
$$= s^3 + 3s^2 + 4s + 12 = 0$$

Routh array:

s^3	1	4
s^2	$\cancel{3}$ 1	$\cancel{12}$ 4
s^1	$\frac{-(4-4)}{1} = \cancel{0}$ 2	$\cancel{0}$ 0
s^0	$\frac{-(0-8)}{2} = 4$	0

$P(s) = s^2 + 4$

$\frac{dP(s)}{ds} = 2s$

Row of zeros indicates even polynomial factor.

No sign changes from rows s^2 to s^0 → no RHP in even polynomial

↳ Thus, no LHP poles due to symmetry.

↳ Both poles of the even polynomial are on the $j\omega$-axis.

No sign changes from rows s^3 to s^2 → last pole must be in the LHP.

$\boxed{\text{1 pole in LHP; 2 poles on imaginary axis; 0 poles in RHP}}$

Check with Matlab: sys = ss(A, B, C, D)

pzmap(sys)

2 Let $G_1(s) = \frac{k_1(s+2)}{(s+3)}$, $G_2(s) = \frac{k_2}{s(s+4)}$, $H(s) = 1$

From Figure 1: $C = G_2[D + G_1(R - HC)]$

(Simplify by $X(s) \to X$)

$$= G_2 D + G_2 G_1 R - G_2 G_1 H C$$
$$\hookrightarrow C(1 + G_2 G_1 H) = G_2 D + G_2 G_1 R$$
$$\hookrightarrow C = \frac{G_2}{1 + G_2 G_1 H} D + \frac{G_2 G_1}{1 + G_2 G_1 H} R$$

(a) Find $\frac{E(s)}{R(s)}$ and $\frac{E(s)}{D(s)}$

$$E = R - C = \left(1 - \frac{G_2 G_1}{1 + G_2 G_1 H}\right) R + \frac{-G_2}{1 + G_2 G_1 H} D = \frac{1 + (H-1) G_2 G_1}{1 + G_2 G_1 H} R - \frac{G_2}{1 + G_2 G_1 H} D$$

$$\frac{E(s)}{R(s)} = \frac{1 + (H-1) G_2 G_1}{1 + G_2 G_1 H} = \frac{1 + (0)}{1 + \frac{k_2}{s(s+4)} \cdot \frac{k_1(s+2)}{(s+3)}(1)} = \frac{s(s+4)(s+3)}{s(s+4)(s+3) + k_1 k_2 (s+2)}$$

$$\frac{E(s)}{D(s)} = \frac{-\frac{k_2}{s(s+4)}}{1 + \frac{k_2}{s(s+4)} \frac{k_1(s+2)}{(s+3)}(1)} = \frac{-k_2(s+3)}{s(s+4)(s+3) + k_1 k_2 (s+2)} = \frac{-G_2}{1 + G_2 G_1 H}$$

2 CONTINUED...

(b) $D(s) = \mathcal{L}\{u(t)\} = \frac{1}{s}$ $\qquad \frac{E(s)}{D(s)} = \frac{-k_2(s+3)}{s(s+4)(s+3)+k_1k_2(s+2)}$

$$e_{ss} = \lim_{s\to 0} s\,\frac{E(s)}{D(s)}D(s) = \frac{E(s)}{D(s)}\left(\frac{s}{s}\right) = \frac{-k_2(s+3)}{s(s+4)(s+3)+k_1k_2(s+2)} = \frac{-3k_2}{2k_1k_2} = \frac{-3}{2k_1}$$

Want $|e(t)| = \left|\frac{-3}{2k_1}\right| < 10^{-4} \rightarrow \frac{1}{k_1} < \left(\frac{2}{3}\right)10^{-4} \rightarrow \boxed{k_1 > 1.5\times 10^{4}}$

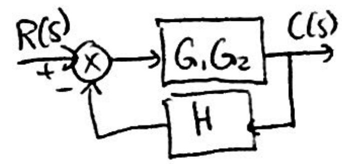
(c) $R(s) = \mathcal{L}\{u(t)\} = \frac{1}{s^2}$ $\qquad \frac{E(s)}{R(s)} = \frac{s(s+4)(s+3)}{s(s+4)(s+3)+k_1k_2(s+2)}$

$$e_{ss} = \lim_{s\to 0} s\,\frac{E(s)}{R(s)}R(s) = \frac{E(s)}{R(s)}\left(\frac{1}{s}\right) = \frac{\cancel{s}(s+4)(s+3)}{s(s+4)(s+3)+k_1k_2(s+2)} = \frac{(4)(3)}{k_1k_2(2)} = \frac{6}{k_1k_2}$$

want $|e(t)| = \left|\frac{6}{k_1k_2}\right| < 10^{-4} \longrightarrow \frac{\cancel{1}}{k_1k_2} < \left(\frac{1}{6}\right)10^{-4} \rightarrow \frac{1}{k_2} < \left(\frac{1}{6}\right)10^{-4}k_1 \rightarrow \frac{1}{k_2} < \frac{1}{4} \rightarrow \boxed{k_2 > 4}$

3 STEADY STATE ERROR FOR UNITY FEEDBACK

(a) $G_1(s) = 2,\quad G_2(s) = \frac{10(s+10)}{s(s+2)},\quad H(s) = s+1,\quad D(s) = 0$

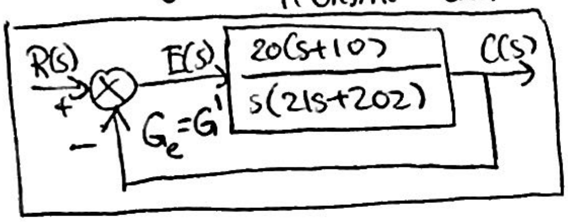


Let $G(s) = \frac{20(s+10)}{s(s+2)}$ (this is from G_1G_2)

and $H(s) = s+1$

Then $G_e(s) = \frac{G(s)}{1+G(s)H(s)-G(s)} = \frac{\frac{20(s+10)}{s(s+2)}}{1+\frac{20(s+10)(s+1)}{s(s+2)}-\frac{20(s+10)}{s(s+2)}} = \frac{20(s+10)}{s(s+2)+20(s+10)(s+1)-20(s+10)}$

$$= \frac{20(s+10)}{s[(s+2)+20(s+10)]} = \boxed{\frac{20(s+10)}{s[21s+202]}}$$



(b) $E(s) = R(s) - C(s)$, $C(s) = G'(s)E(s)$ $\} \rightarrow E(s)[1+G'(s)] = R(s) \rightarrow E(s) = \frac{1}{1+G'(s)}R(s)$

From (a), $G'(s) = \frac{20(s+10)}{s[21s+202]}$ (also $G_e(s)$)

Then $E(s) = \frac{1}{1+\frac{20(s+10)}{s[21s+202]}} = \boxed{\frac{s(21s+202)}{s(21s+202)+20(s+10)}}$

(c) From G', we see that the denominator is $s(21s+202)$, so the system is $\boxed{\text{Type 1}}$

(d) $K_p = \lim_{s\to 0} G'(s) = \frac{20(10)}{0} = \boxed{\infty}$, $K_v = \lim_{s\to 0} sG'(s) = \frac{20(s+10)}{(21s+202)} = \frac{200}{202} = \boxed{\frac{100}{101}}$, $K_a = \lim_{s\to 0} s^2G'(s) = \boxed{0}$

(e) STEP: $e(\infty) = \frac{1}{1+K_p} = \boxed{0}$, RAMP: $e(\infty) = \frac{1}{K_v} = \boxed{\frac{101}{100}}$, PARABOLIC: $e(\infty) = \frac{1}{K_a} = \boxed{\infty}$

Q4. $\dot{x} = Ax + Bu = \begin{bmatrix} -9 & -5 & -1 \\ 1 & 0 & -2 \\ -3 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} r$

$$y = [1 \ -2 \ 4] x$$

a.) from equation 7.89 in Nise

$$\lim_{s \to 0} sE(s) = \cancel{\lim_{s\to 0} sR(s)\ldots} \quad e(\infty) = 1 - y_{ss}$$

$$e(\infty) = 1 + CA^{-1}B$$

$$A = \begin{bmatrix} -9 & -5 & -1 \\ 1 & 0 & -2 \\ -3 & -2 & -5 \end{bmatrix} \qquad A^{-1} = \frac{1}{17} \begin{bmatrix} 4 & 23 & -10 \\ -11 & -42 & 19 \\ 2 & 3 & -5 \end{bmatrix}$$

$$CA^{-1}B = [1 \ -2 \ 4] \begin{bmatrix} 4 & 23 & -10 \\ -11 & -42 & 19 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \times \frac{1}{17}$$

$$= [1 \ -2 \ 4] \begin{bmatrix} 37 \\ -72 \\ -7 \end{bmatrix} \times \frac{1}{17} = \frac{1}{17} [37 + 144 - 28]$$

$$= \frac{153}{17} = 9 \quad \therefore \boxed{e(\infty) = 9+1} \Rightarrow \boxed{e(\infty) = 10}$$

b.) from eqn 7.103 in Nise the steady state error of an system with unit ramp input.

$$e(\infty) = \lim_{t \to \infty} (t - y_{ss}) = \lim_{t \to \infty} [(1 + CA^{-1}B)t + C(A^{-1})^2 B]$$

$$[(1 + CA^{-1}B]t + C(A^{-1})^2 B]$$

$$\downarrow \qquad\qquad \downarrow$$

$$10 \qquad\qquad C \times (A^{-1})^2 \times B$$

$$(A^{-1}) = \frac{1}{17}\begin{bmatrix} 4 & 23 & -10 \\ -11 & -42 & 19 \\ 2 & 3 & -5 \end{bmatrix} \qquad (A^{-1})^2 = \frac{1}{17^2}\begin{bmatrix} -257 & -904 & 447 \\ 456 & 1568 & -783 \\ -35 & -95 & 62 \end{bmatrix}$$

$$C \times (A^{-1})^2 \times B = \frac{1}{17^2}\begin{bmatrix} 1 & -2 & 4 \end{bmatrix}\begin{bmatrix} -257 & -904 & 447 \\ 456 & 1568 & -783 \\ -35 & -95 & 62 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$= \frac{1}{17^2}\begin{bmatrix} 1 & -2 & 4 \end{bmatrix}\begin{bmatrix} -514 - 2712 + 1788 \\ 912 + 4704 - 3132 \\ -70 - 285 + 248 \end{bmatrix}$$

$$= \frac{1}{17^2}\begin{bmatrix} 1 & -2 & 4 \end{bmatrix}\begin{bmatrix} -1438 \\ + \cancel{1750}\ 2484 \\ -107 \end{bmatrix} = \frac{-6834}{17^2}$$

$$= -23.64$$

$$\therefore \quad e(\infty) = \lim_{t\to\infty} (10t + (-23.64))$$

$$= \lim_{t\to\infty} 9t - 23.64$$

$$\therefore \boxed{e(\infty) = \infty}$$

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5 (a) $G(s) = \dfrac{k(s+20)}{(s+5)(s+15)}$

i) ① $\dfrac{1}{1+G(s)} = \dfrac{(s+5)(s+15)}{(s+5)(s+15)+k(s+20)}$ → 2 closed loop poles → 2 banches

② Poles appear as complex conjugate pairs (symmetry about real axis)

③  Open loop poles & zeros. Real axis segments: $(-\infty, -20)$ $(-15, -5)$

④ Root locus begins @ open loop poles $s=-15, s=-5$
ends @ $-\infty$ and $s=-20$

⑤ $\theta_a = \dfrac{-(1)\pi}{(2-1)} = -\pi$ ... asymptote must be @ angle π
$\sigma_a = \dfrac{(-5-15)-(-20)}{2-1} = 0$ and asymptote intersects @ $s<0$

⑥ $\dfrac{1}{\sigma+20} = \dfrac{1}{\sigma+5} + \dfrac{1}{\sigma+15} = \dfrac{\sigma+15+\sigma+5}{(\sigma+5)(\sigma+15)} = \dfrac{2\sigma+20}{(\sigma+5)(\sigma+15)}$

$0 = (\sigma+5)(\sigma+15) - (\sigma+20)(\sigma+10)(2)$

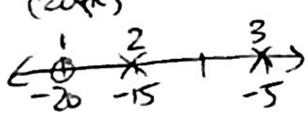
$0 = -\sigma^2 - 40\sigma - 325 \rightarrow \sigma_1 = -5(4+\sqrt{3})$ or -28.66, $\sigma_2 = 5(\sqrt{3}-4)$ or -11.34

Break away location @ $s = 5(\sqrt{3}-4)$ or -11.34
Break in location @ $s = -5(4+\sqrt{3})$ or -28.66

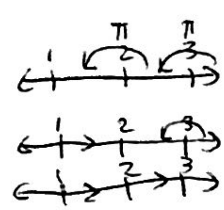
⑦ Denominator of closed loop TF: $(s+5)(s+15)+k(s+20) = s^2+(20+k)s+(75+20k)$

s^2	1	$(75+20k)$
s^1	$(20+k)$	0
s^0	$\frac{(20+k)(75+20k)}{(20+k)}$	0

No positive solutions of k make the rows zeros → no crossing over $j\omega$-axis.

⑧ For 

$\theta_1 - \theta_2 - \theta_3 = 180$



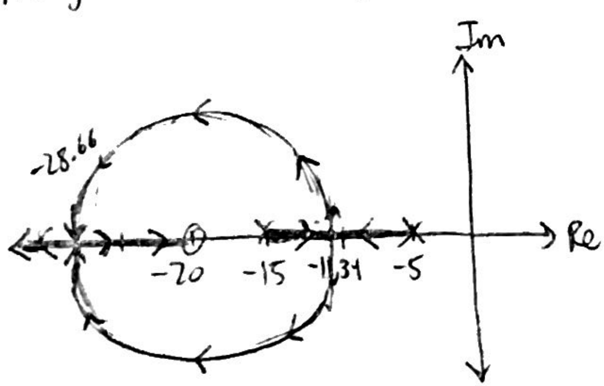
Angle of arrival θ_1: $\theta_1 - \pi - \pi = \pi \rightarrow \theta_1 = 3\pi = \pi$

Angle of dep. θ_2: $0 - \theta_2 - \pi = \pi \rightarrow \theta_2 = 2\pi = 0$

Angle of dep. θ_3: $0 - 0 - \theta_3 = \pi \rightarrow \theta_3 = \pi$

ii) No $j\omega$-axis intercepts for $k \geq 0$

iii)



iv) Stable for all $k \geq 0$

v) See attached plot.

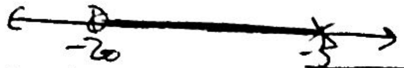
HW5 EE128

5 (b) $G(s) = \dfrac{k(s+20)}{(s^2+10s+50)(s+5)}$

① $\dfrac{1}{1+G(s)} = \dfrac{(s^2+10s+50)(s+5)}{(s^2+10s+50)(s+5)+k(s+20)} \rightarrow$ 3 closed loop poles $\rightarrow$ 3 branches

② Symmetry about real axis

③ Open loop poles: $s = -5, -5-5i, -5+5i$
Open loop zeros: $s = -20$



Real axis segment: $\boxed{(-20, -5)}$

④ Root locus begins @ open loop poles $s = -5, -5-5i, -5+5i$
ends @ $s = -20$, two branches end on infinite zeros

⑤ $\theta_a = \dfrac{-\pi}{3-1} = \dfrac{-\pi}{2}$ and for $l=1$, $\theta_a = \dfrac{-3\pi}{3-1} = \dfrac{-3\pi}{2}$

For asymptotes @ $\boxed{\text{angles } \frac{\pi}{2} \text{ and } \frac{3\pi}{2}}$, intersects Real axis @ $\dfrac{(-5-5-5i-5+5i)-(-20)}{(3-1)} = \boxed{\dfrac{5}{2}}$

⑥ From ③ we see there are no break away / break in points, as real axis open loop pole $s = -5$ begins and ends on the real axis (ending @ zero $s = -20$)

⑦ Denominator of closed loop TF: $(s^2+10s+50)(s+5) + k(s+20) = s^3 + 15s^2 + (100+k)s + (250+20k)$

s^3	1	$(100+k)$
s^2	15	$(250+20k)$
s^1	$\dfrac{-(250+20k)+(100+k)15}{15}$	0
s^0	$250+20k$	0

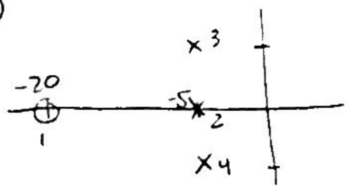
$\boxed{K=250}$ makes s^1 row zeros.

$15s^2 + (250+20k)\big|_{k=250} = 15s^2 + 21(250) = 0$

$\hookrightarrow \boxed{s = \pm 5i\sqrt{14} = \pm 18.7i}$

The root locus $\boxed{\text{crosses the } j\omega\text{-axis @ } \pm j18.7 \text{ at gain } 250}$

⑧



$\theta_1 - \theta_2 - \theta_3 - \theta_4 = \pi$

θ_1: $\theta_1 - (\pi + \tan^{-1}(\frac{1}{3})) - (\pi - \tan^{-1}(\frac{1}{3})) - \pi = \pi \rightarrow \theta_1 = 0°$

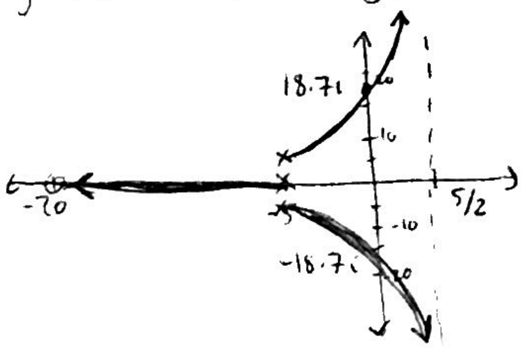
θ_2: $0 - \theta_2 - (-\frac{\pi}{2}) - (\frac{\pi}{2}) = \pi \rightarrow \theta_2 = -\pi$

θ_3: $\tan^{-1}(\frac{1}{3}) - (\frac{\pi}{2}) - \theta_3 - (\frac{\pi}{2}) = \pi \rightarrow \theta_3 = \tan^{-1}(\frac{1}{3}) \approx 18.4°$

θ_4: $-\tan^{-1}(\frac{1}{3}) - (-\frac{\pi}{2}) - (-\frac{\pi}{2}) - \theta_4 = \pi \rightarrow \theta_4 = -\tan^{-1}(\frac{1}{3})$

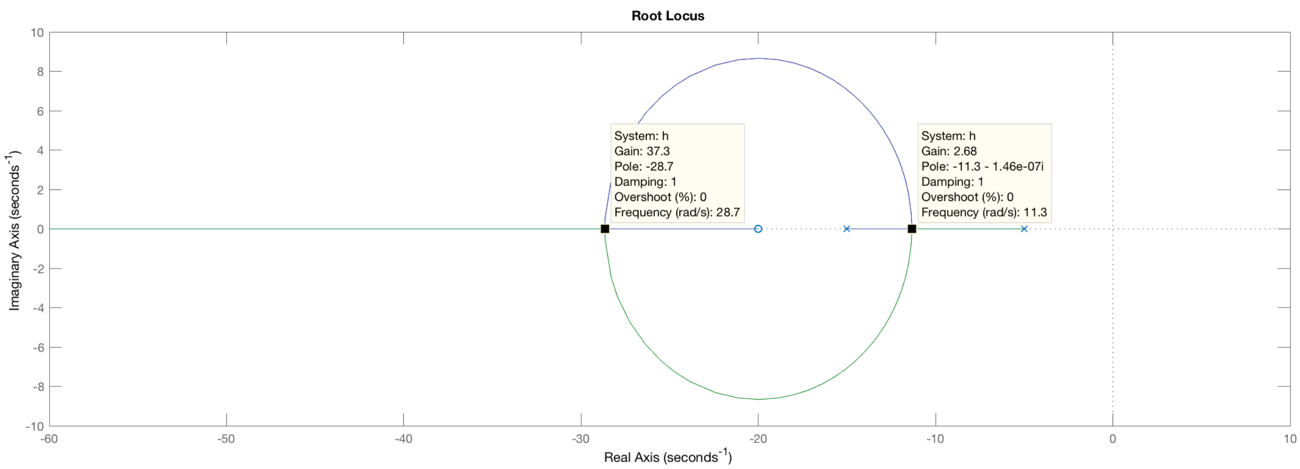
i:) $j\omega$-axis is crossed @ $\pm j18.7$ with gain $k = 250$

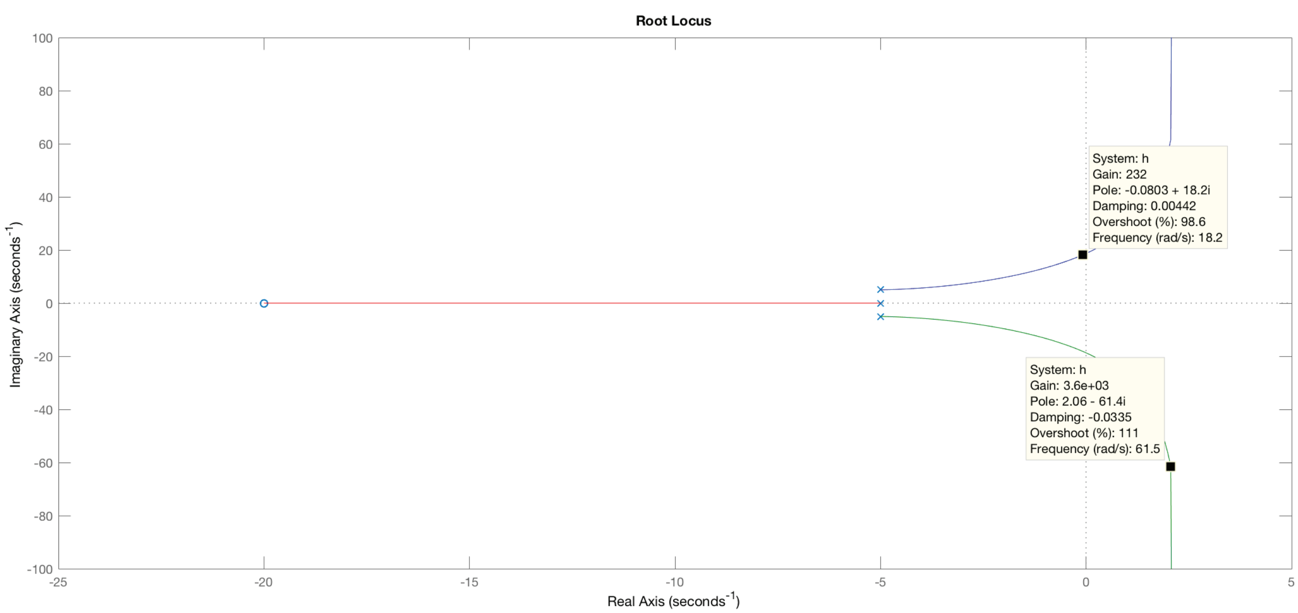
iii.)



iv) Stable for $k \in [0, 250)$

v) See attached plot.


Root Locus
System: h
Gain: 37.3
Pole: -28.7
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 28.7
System: h
Gain: 2.68
Pole: -11.3 - 1.46e-07i
Damping: 1
Overshoot (%): 0
Frequency (rad/s): 11.3
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})


Root Locus
System: h
Gain: 232
Pole: -0.0803 + 18.2i
Damping: 0.00442
Overshoot (%): 98.6
Frequency (rad/s): 18.2
System: h
Gain: 3.6e+03
Pole: 2.06 - 61.4i
Damping: -0.0335
Overshoot (%): 111
Frequency (rad/s): 61.5
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})