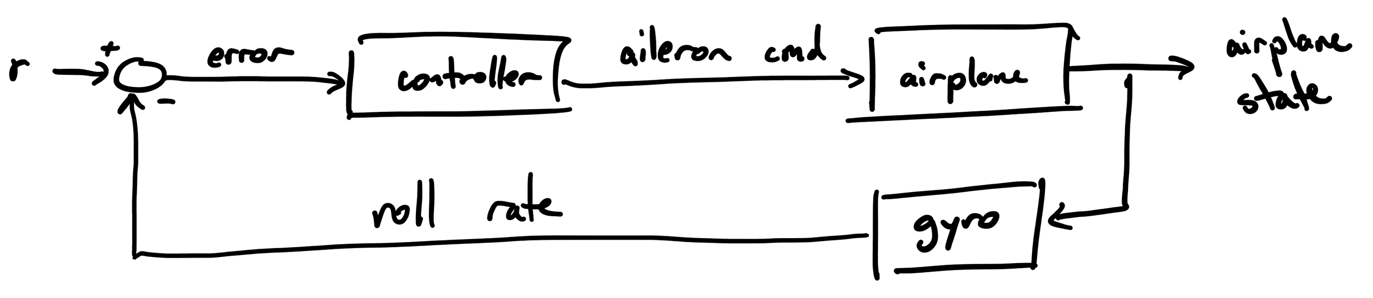
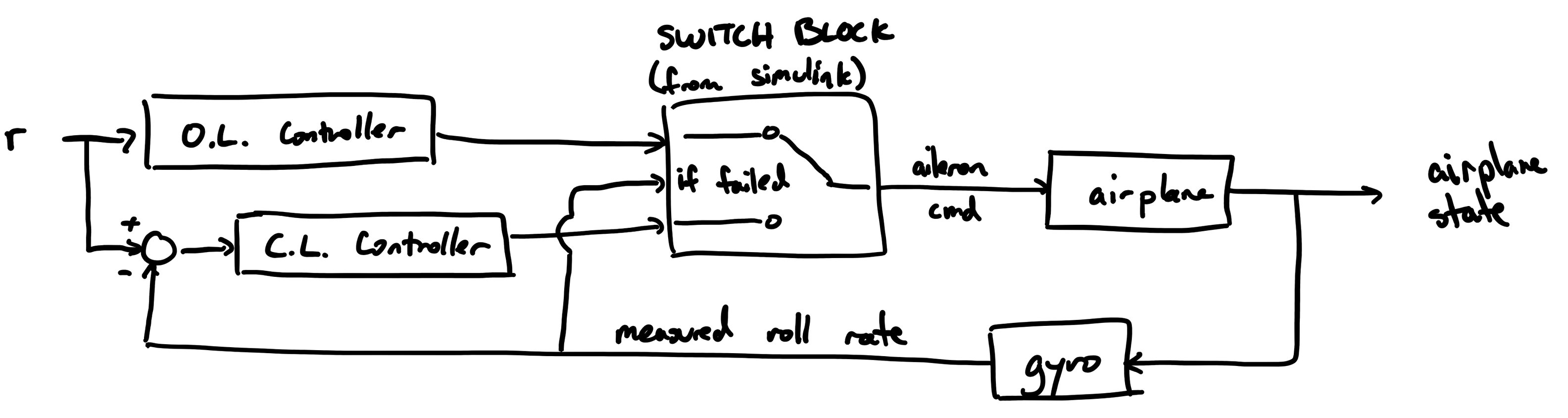
Problem Set 1

Tuesday, August 27, 2019 8:52 AM

1 a)



b)



2 a)

$$g(\varepsilon) = 1000 \log(\varepsilon + 1)$$
$$= \left[1000 \log(a+1) + 1000 \frac{1}{a+1}(\varepsilon - a) + \cdots \right]\Big|_{a=0} \quad \text{(about } \varepsilon = 0 \text{ b/c } |x(t)| \ll 1)$$
$$= 0 + 1000\varepsilon + \cdots$$
$$\approx 1000\varepsilon$$

$$y(t) = \delta(t) + g(\varepsilon(t))$$
$$y(t) = \cancel{\delta(t)}^{\,0} + g(x(t) - k y(t))$$
$$y(t) = g\left(x(t) - \tfrac{1}{3} y(t)\right)$$
$$y(t) \approx 1000\left(x(t) - \tfrac{1}{3} y(t)\right)$$
$$\left(1 + \tfrac{1000}{3}\right) y(t) \approx 1000\, x(t)$$
$$y(t) \approx \frac{3000}{1003} x(t) \approx 3x(t) \checkmark$$

b)

$$y_1 = 3$$
$$3 = g\left(x_1 - \tfrac{1}{3}\cdot 3\right)$$
$$3 = g(x_1 - 1)$$
$$3 = 1000 \log(x_1)$$
$$x_1 = e^{3/1000} = 1.003$$

c)

$$y_2 = 6$$
$$6 = g\left(x_2 - \tfrac{1}{3}\cdot 6\right)$$
$$6 = g(x_2 - 2)$$
$$6 = 1000 \log(x_2 - 1)$$
$$x_2 = e^{6/1000} + 1 = 2.006$$

d) 0.3% for both

3 i)

$$H_1(s) = \frac{1}{s^2 + 14s + 48} = \frac{1}{(s+6)(s+8)} = \frac{1/2}{s+6} - \frac{1/2}{s+8}$$
$$h_1(t) = \frac{1}{2} e^{-6t} - \frac{1}{2} e^{-8t} \quad \text{for } t > 0$$

ii)

$$H_2(s) = \frac{s}{s^2 + 14s + 48} = \frac{s}{(s+6)(s+8)} = \frac{-3}{s+6} + \frac{4}{s+8}$$
$$h_2(t) = -3e^{-6t} + 4e^{-8t} \quad \text{for } t > 0$$

iii)

$$H_3(s) = \frac{s+10}{s^2 + 14s + 48} = \frac{s+10}{(s+6)(s+8)} = \frac{2}{s+6} - \frac{1}{s+8}$$
$$h_3(t) = 2e^{-6t} - e^{-8t} \quad \text{for } t > 0$$

iv)

$$H_4(s) = \frac{1}{s^2 + 6s + 18} = \frac{\frac{1}{3}(3)}{(s+3)^2 + 3^2}$$
$$h_4(t) = \frac{1}{3} e^{-3t} \sin(3t) \quad \text{for } t > 0$$

$$\begin{cases} A + B + C = 0 \\ 14As + 8Bs + 6Cs = 0 \\ 48A = 1 \end{cases}$$

v)

$$H_5(s) = \frac{1}{s^3 + 14s^2 + 48s} = \frac{1}{s(s+6)(s+8)} = \frac{A}{s} + \frac{B}{s+6} + \frac{C}{s+8} = \frac{1/48}{s} - \frac{1/12}{s+6} + \frac{3/48}{s+8}$$
$$h_5(t) = \frac{1}{48} - \frac{1}{12} e^{-6t} + \frac{3}{48} e^{-8t} \quad \text{for } t > 0$$

4 Using Final Value Theorem and Initial Value Theorem:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) \qquad \lim_{t\to 0^+} f(t) = \lim_{s\to\infty} sF(s)$$

i)

$$Y_1(s) = \frac{s}{s+5}$$
$$Y_1(s) = 1 - \frac{5}{s+5} \iff y_1(t) = \delta(t) - 5e^{-5t} \quad \text{for } t > 0 \quad \text{so} \quad \lim_{t\to 0^+} y_1(t) = -5$$
$$\lim_{t\to\infty} y_1(t) = \lim_{s\to 0} \frac{s^2}{s+5} = 0$$

ii)

$$Y_2(s) = \frac{s-3}{s+5}$$
$$Y_2(s) = 1 - \frac{8}{s+5} \iff y_2(t) = \delta(t) - 8e^{-5t} \quad \text{for } t > 0 \quad \text{so} \quad \lim_{t\to 0^+} y_2(t) = -8$$
$$\lim_{t\to\infty} y_2(t) = \lim_{s\to 0} \frac{s^2 - 3s}{s+5} = 0$$

iii)

$$Y_3(s) = \frac{(s+3)}{s(s+5)}$$
$$\lim_{t\to 0^+} y_3(t) = \lim_{s\to\infty} \frac{s(s+3)}{s(s+5)} = \lim_{s\to\infty} \frac{1}{1} = 1$$
$$\lim_{t\to\infty} y_3(t) = \lim_{s\to 0} \frac{s(s+3)}{s(s+5)} = \frac{3}{5}$$

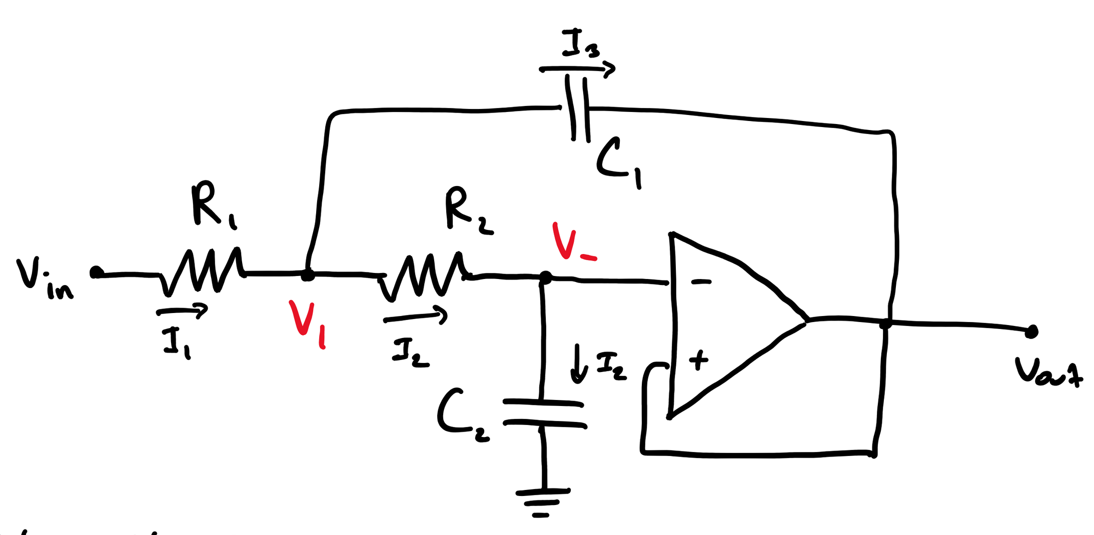
iv)

$$Y_4(s) = \frac{1}{s(s+5)}$$
$$\lim_{t\to 0^+} y_4(t) = \lim_{s\to\infty} \frac{s}{s(s+5)} = 0$$
$$\lim_{t\to\infty} y_4(t) = \lim_{s\to 0} \frac{s}{s(s+5)} = \frac{1}{5}$$

v)

$$Y_5(s) = \frac{(s+3)^2}{(s+5)^2}$$
$$Y_5(s) = 1 - \frac{4(s+4)}{(s+5)^2} \iff y_5(t) = \delta(t) + 4e^{-5t}(t-1) \quad \text{for } t > 0 \quad \text{so} \quad \lim_{t\to 0^+} y_5(t) = -4$$
$$\lim_{t\to\infty} y_5(t) = \lim_{s\to 0} \frac{s(s+3)^2}{(s+5)^2} = 0$$

5



$$V_{in} = V_1 + R_1 I_1$$
$$V_{in} = V_1 + R_1(I_2 + I_3)$$
$$V_1 - V_{out} = \frac{1}{C_1 s} I_3 \Rightarrow I_3 = C_1 s (V_1 - V_{out})$$
$$V_- = \frac{1}{C_2 s} I_2 \Rightarrow I_2 = C_2 s V_-$$
$$V_1 = V_- + R_2 I_2$$
$$V_{in} = V_- + R_2 I_2 + R_1\left(C_1 s (V_1 - V_{out}) + C_2 s V_-\right)$$
$$V_{in} = V_- + R_2 C_2 s V_- + R_1\left(C_1 s (V_- + R_2 C_2 s V_- - V_{out}) + C_2 s V_-\right)$$
$$V_{in} = V_- + R_2 C_2 s V_- + R_1 C_1 s V_- + R_1 C_1 s R_2 C_2 s V_- - R_1 C_1 s V_{out} + R_1 C_2 s V_-$$
$$V_{in} = \left(1 + R_2 C_2 s + \cancel{R_1 C_1 s} + R_1 R_2 C_1 C_2 s^2 - \cancel{R_1 C_1 s} + R_1 C_2 s\right) V_{out}$$

$$H(s) = \frac{1}{1 + R_1 C_2 s + R_2 C_2 s + R_1 R_2 C_1 C_2 s^2}$$