PS2 prob 4.

$K = K_S + K_A$, $x_1 = x$, $\dot{x}_1 = x_2 = \dot{x}$, and $V_0 = \left(\sqrt{\frac{K}{\epsilon_0 A}}\right) x_0$

a) $F_A = m_A \ddot{x} + Kx + b_S \dot{x} = m_A \dot{x}_2 + Kx_1 + b_S x_2$

solve for $\dot{x}_2$:

$$\dot{x}_2 = \frac{1}{m_A}\left[F_A - Kx_1 - b_S x_2\right]$$

$$F_A = \frac{\epsilon_0 A}{x} V^2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-K}{m_A} & \frac{-b_S}{m_A} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_A} \end{bmatrix} F_A(x, u)$$

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b). Linearize $F_A()$ at x_0, V_0

$$F_A(x_0 + \delta x, V_0 + \delta V) \approx F_A(x_0, V_0) + \frac{\partial F_A}{\partial x}\delta x + \frac{\partial F_A}{\partial V}\delta V$$

$$F_A(x_0, V_0) = \frac{\epsilon_0 A}{x_0} V_0^2 = \frac{\epsilon_0 A}{x_0} \cdot x_0^2 \left(\frac{K}{\epsilon_0 A}\right) = Kx_0$$

$$\left.\frac{\partial F_A}{\partial V}\right|_{x_0, V_0} = \frac{\epsilon_0 A}{x_0}(2V_0) \quad , \quad \left.\frac{\partial F_A}{\partial x}\right|_{x_0, V_0} = \frac{-\epsilon_0 A}{x_0^2} V_0^2 = \frac{-\epsilon_0 A}{x_0^2} \cdot x_0^2 \left(\frac{K}{\epsilon_0 A}\right)$$

$$= -K$$

$$= \frac{2\epsilon_0 A}{x_0} \cdot x_0 \sqrt{\frac{K}{\epsilon_0 A}}$$

$$= 2\sqrt{K\epsilon_0 A}$$

Finally, $F_A(x_0 + \delta x, V_0 + \delta V) = Kx_0 - K\delta x + 2\sqrt{K\epsilon_0 A}\,\delta V$

since $\delta x = x_1 - x_0$, the A_{22} term changes to $-2K$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{-2K}{m_A} & \frac{-b_S}{m_A} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m_A \end{bmatrix} \left(2Kx_0 + 2\sqrt{K\epsilon_0 A}\,\delta V\right)$$