Problem 1.

\[
\begin{align*}
\dot{i}_1 &\rightarrow x_1 & L_1 &\rightarrow M_1 & L_2 &\rightarrow M_2 \\
\dot{i}_2 &\rightarrow x_2 & C_1 &\rightarrow k_1 = \frac{1}{C_1} & V_1 &\rightarrow F_1 \\
R &\rightarrow B & V_2 &\rightarrow F_2 \\
\end{align*}
\]

Note \( V_1, L_1, C_1 \) all have same current \( i_1 = i_2 \). Also \( R \frac{L_2}{V_2} \) have current \( i_2 = i_2 \).
(a) \[ F_A - F_{AV} = F_{Ax} = F_{Ay} = 0 \]
\[ F_A = k_A x - k_x x - b_x \dot{x} = M_A \ddot{x} \]
\[ -M_A \ddot{x} + b_x \dot{x} - k_x x + F_A = 0 \]
\[ \ddot{x} = \frac{1}{M_A} \begin{bmatrix} 0 & 1 \\ -b_x & k_x \end{bmatrix} \begin{bmatrix} x \\ V \end{bmatrix} + F_A(x, V) \]

\[ s_0 \begin{bmatrix} \dot{x} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} \frac{b_x}{M_A} + \frac{k_x}{M_A} \\ \frac{k_x}{M_A} \end{bmatrix} s_0 \begin{bmatrix} x \\ V \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F_A(x, V) \]

(b) \[ F_A(x, V) = \frac{\xi_A}{\xi_0} \frac{A}{x_0} V^2 \]
\[ F_A(x, V) = F_{Ax}(x, V) + F_{AV}(x, V) \]
\[ F_{Ax}(x, V) = \frac{1}{\xi_0} \frac{b_x}{\xi_0} \frac{2}{x_0} \frac{A}{x_0} V^2 \]
\[ \dot{F}_{AV}(x, V) = \frac{1}{\xi_0} \frac{b_x}{\xi_0} \frac{1}{x_0} \frac{A}{x_0} V \]
\[ F_A(x, V) = \frac{2}{\xi_0} \frac{b_x}{\xi_0} \frac{A}{x_0} V \]
\[ \dot{F}_A(x, V) = \frac{2}{\xi_0} \frac{b_x}{\xi_0} \frac{A}{x_0} V \]

\[ \dot{x} = \begin{bmatrix} 0 & 1 \\ -b_x & k_x \end{bmatrix} \begin{bmatrix} x \\ V \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} F_A(x, V) \]

(b) **Jacobian Alternative**
\[ \dot{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \]
\[ \dot{A}(t) = A \dot{s}_0(t) + B \dot{F}_A(t) \]
\[ A = \begin{bmatrix} \frac{\xi_A}{\xi_0} & \frac{A}{x_0} \\ \frac{2}{\xi_0} & \frac{A}{x_0} \end{bmatrix} \]
\[ B = \begin{bmatrix} 0 \\ \frac{2}{\xi_0} \frac{b_x}{\xi_0} \frac{A}{x_0} \end{bmatrix} \]
\[ \dot{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{\xi_A}{\xi_0} & \frac{A}{x_0} \\ \frac{2}{\xi_0} & \frac{A}{x_0} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2}{\xi_0} \frac{b_x}{\xi_0} \frac{A}{x_0} \end{bmatrix} V \]
a) \[ X(s) = \frac{U(s)}{S^3 + 12S^2 + 45S + 50} \]

\[ L^{-1}\{X(s)\} = U(t) - 12 \frac{d^2X}{dt^2} - 45 \frac{dX}{dt} - 50X \]

\[ \dot{X} = AX + Bu \]

\[ X = [\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -50 & -45 & -12 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \]

\[ Y(s) = (S^3 + S^2) X(s) \]

\[ y(t) = L^{-1}\{Y(s)\} = \frac{d^3X}{dt^3} + \frac{d^2X}{dt^2} \]

\[ y(t) = Cx + Du \]

\[ A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -50 & -45 & -12 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \]

\[ C = \begin{bmatrix} -1 & -45 & -50 \end{bmatrix}, \quad D = 1 \]

b) \[ y(t) = U(t) - 11 \frac{d^2x}{dt^2} - 45 \frac{dX}{dt} - 50X \]

\[ x(t) = [\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}] \]

\[ \frac{d^3x(t)}{dt^3} \]