Due at 1700, Fri. Oct. 4 in gradescope

Note: up to 2 students may turn in a single writeup. Reading Nise 7.8.

1. (20 pts) Routh Array (Nise 6.3)
   In the control system in Fig. 1, \( D(s) = 0 \), \( G_1(s) = k \), \( H(s) = \frac{1}{s+2} \), and
   \[
   G_2(s) = \frac{s + 1}{s^3 + 2s^2 + 5s}
   \]
   [2pts] a. Determine the closed loop transfer function \( \frac{C(s)}{R(s)} \).
   [16pts] b. Using the Routh-Hurwitz table, find the range of \( k \) for the system to have all closed loop poles in the LHP.
   [2pts] c. Find the value of \( k \) that makes the system have poles on the \( j\omega \) axis.

2. (20 pts) Steady state error (Nise 7)
   Consider a PI controller \( G_1(s) = k_p + k_i/s \) for plant \( G_2(s) = \frac{1}{s+1} \) and \( H(s) = 1 \), with \( r(t) = 0 \).
   [4pts] a. For a step disturbance \( d(t) \), show that the steady state error is zero.
   [8pts] b. For a disturbance input \( d(t) = \cos(\omega_n t)u(t) \), (e.g. a power line signal) find the steady state error \( e(t) \) after all transients have died away. (Note \( s = 0 \) is not part of region of convergence.) Hint: consider response of system to \( e^{j\omega_n t} \), and let \( s = j\omega_n \).
   [8pts] c. For the same disturbance in b., find a new \( G_1(s) \) which will have zero steady state error for a step disturbance, and zero steady state error for \( d(t) = \cos(\omega_n t)u(t) \). (Hint: add something else in the controller which copies the disturbance.)

3. (20 pts) Steady state error for non-unity feedback (Nise 7.4,7.6)
   For the system in Fig. 1, let \( G_1(s) = 12 \), \( G_2(s) = \frac{s+3}{s^3+3s+4} \) and \( H(s) = \frac{s+1}{s} \), \( D(s) = 0 \). \( E = R - C \).
   [5pts] a. Find and draw the equivalent unity gain system.
   [3pts] b. Find \( E(s) \).
   [3pts] c. What is the system type?
   [4pts] d. What are static error constants \( K_p, K_i, K_d \) for step and ramp inputs?
   [5pts] e. What is the steady state error for a unit step input? For a unit ramp input? For \( r(t) = t^2u(t) \)?

4. (20 pts) Steady state error (Nise 7.8)
   [10pts] a) Find steady state error for \( r(t) \) a unit step input, using input substitution.
   [10pts] b) Find steady state error for \( r(t) \) a unit ramp input, using input substitution.
   Given system:
   \[
   \dot{x} = Ax + Bu = \begin{bmatrix} -10 & -5 & -1 \\ 1 & 0 & -2 \\ -3 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} r, \text{ and } y = [1 & -2 & 4]x
   \]

5. (20 pts) Closed-loop poles from geometry (Nise 8.3)
   For each part below with open loop transfer function \( G_i(s) \) in unity gain feedback (Fig.2):
   [5pts] a) Showing pole-zero sketch with angles, find \( j\omega \) axis intercepts.
   Verify that \( \angle G_i(j\omega) = (2k + 1)180^\circ \).
   [4pts] iii) Show in a sketch length of vectors to \( j\omega \) axis intercepts, and find \( k \).
   Verify that \( |kG_i(j\omega)| = 1 \).
   [11 pts] i) Verify the values found using MATLAB root locus function and include plot.

   a) \( G_1(s) = \frac{k}{(s+6)(s+4)(s-2)} \) b) \( G_2(s) = \frac{k(s+4)}{(s+6)(s-2)} \)

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Fig. 1. Control System Block Diagram. Fig. 2. Unity Gain Feedback.