

# Lab 2: Basic Concepts in Control System Design

“There is nothing worse than a sharp image of a fuzzy concept.” – Ansel Adams

## 1 Objectives

The goal of this lab is to understand some of the basic concepts behind control theory: equilibrium points, stability, feedback, steady-state response, and linearization.

## 2 Theory

### 2.1 Poles and Zeros

Recall that the *poles* of a transfer function (TF) are those values of the Laplace transform variable,  $s$ , for which the denominator of the TF is zero, including any roots shared with the numerator, i.e. roots of the denominator which ‘cancel’ out with the numerator. Similarly, the *zeros* of a transfer function (TF) are those values of  $s$  that cause the TF to become zero, including roots shared with the denominator.

### 2.2 Stability

The time response of a linear time invariant (LTI) dynamical system  $c(t) = c_{\text{forced}}(t) + c_{\text{natural}}(t)$ , where  $c_{\text{forced}}(t)$  is the forced response (driven by the input applied to the system) and  $c_{\text{natural}}(t)$  is the natural response (driven by the system’s initial states).

A linear, time-invariant system is called:

- *stable* if the natural response approaches zero as time approaches infinity.
- *unstable* if the natural response grows without bound as time approaches infinity.
- *marginally stable* if the natural response neither decays nor grows but remains constant or oscillates as time approaches infinity.

For LTI dynamical systems one can discuss stability easily in terms of the locations of the poles of the system’s TF. A system is stable if all poles lie in the left half of the complex plane (LHP). A system is unstable if its TF has at least one pole in the right half of the complex plane (RHP), or a pole of multiplicity greater than one on the imaginary axis. A system is marginally stable if it has no poles in the RHP and only poles of multiplicity one on the imaginary axis.

### 3 Pre-Lab

#### 3.1 Simple feedback system

Consider the feedback system shown in Figure 1.

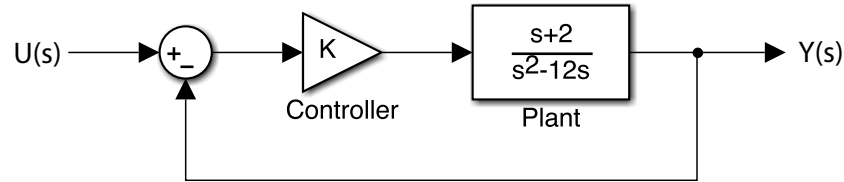


Figure 1: Simple proportional control system

Suppose the control goal is to track a step input.

1. What are the poles and zeros of the open-loop system? Is the open-loop system stable? Can the open-loop system track a step input? Explain.
2. Suppose you use proportional feedback control as shown in Figure 1. Explain *why* the scheme above is called proportional feedback. Explain (mathematically) how this scheme can be used to stabilize the system.
3. Is the system stable for all values of  $K > 0$ ? Prove or find a counterexample.
4. For what values of  $K > 0$  is the system completely oscillatory?
5. For what values of  $K > 0$  is the system stable?

#### 3.2 Nonlinear damped pendulum

The dynamics of a real plant are often nonlinear in nature. There is a plethora of interesting mathematical and physical differences between linear and nonlinear systems. In this part, you study the mathematical differences between linear and nonlinear versions of the pendulum with respect to equilibrium points.

Consider a damped pendulum as in Figure 2.

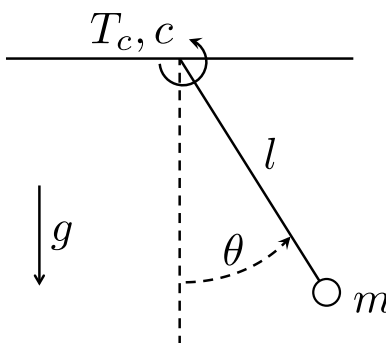


Figure 2: A damped pendulum

The nonlinear equation of motion for the pendulum in Figure 2 is given by:

$$\ddot{\theta} + \frac{c}{ml} \dot{\theta} + \frac{g}{l} \sin \theta = \frac{T_c}{ml^2} \quad (1)$$

Here  $\theta$  is the angle of the pendulum from the vertical (in rad),  $c$  is the velocity damping term (in  $\text{s}^{-1}$ ),  $m$  is the mass of the pendulum (in kg),  $l$  is the length (in m),  $g$  is the acceleration due to gravity (in  $\text{ms}^{-2}$ ) and  $T_c$  is the force input (in  $N \cdot m$ ). For this problem assume that  $g = 9.81 \text{ ms}^{-2}$ ,  $l = 2.0 \text{ m}$ ,  $m = 1.0 \text{ kg}$ , and  $c = 0.25 \text{ s}^{-1}$ .

1. What is the nonlinear term in the pendulum equation above?
2. Very often, you “remove” the nonlinearity in the plant by linearizing the plant about an operating point. In this case, we will do a Taylor series expansion around  $\theta = 0$  for the nonlinearity and disregard the nonlinear terms in the expansion. What are the first three terms in the Taylor series expansion of the nonlinearity? What is the linearized version of the simple pendulum equation?

## 4 Lab

### 4.1 Simple Feedback System

1. Create a Simulink model of the simple feedback system shown in Figure 1.
2. Verify the system is oscillatory for the value(s) of  $K$  that you found in the Pre-Lab. You can test some select values of  $K$  in this interval. Plot the step response of the system (i.e. the input signal should be a step input which steps from 0 to 1 at time  $t_0 = 0$ ) for four different values of  $K$ .
3. Verify the system is stable for values of  $K$  in the corresponding interval you found in the Pre-Lab. You can test some select values in this interval. Plot the step response of the system for four different values of  $K$  in this interval (on the same graph). **Report the location of the poles of the closed loop system for each value of  $K$ .**
4. Suppose we want our output response to not significantly exceed the final value of our input (step) signal. We can set a bound on our response using the maximum percent overshoot (%OS) – the percent by which the maximum value of the response exceeds the steady state (final) value. In this case, let us specify that we want the %OS to be less or equal than 25% for all  $t \geq t_0$ .

*Using simulation* (not math), determine for what values of  $K$  (find a range – precision of 0.1 for bounds is good enough) the %OS is less or equal than 25%? Also show graphically that your bounds on  $K$  are correct (needs to be evident on the plot). Report the location of the closed loop poles for this value of  $K$ .

5. How do the poles of the system change as  $K$  increases?

Notice that in designing a control system, we first analyzed the stability of our open-loop plant. If our open-loop plant is unstable, we use feedback to stabilize the system. Then we pick the values for the parameters in our control law so our control objective (in this case, a constraint on the percent overshoot) is achieved. Of course, there will generally also be other constraints (transient response for example). Satisfying all the design requirements is the goal of control theory.

## 4.2 Nonlinear Damped Pendulum

1. Model the nonlinear damped pendulum in Simulink. Note that this is *not* a feedback control system – you are building a block diagram representation of a plant! Instead of using derivatives, generate the necessary  $\theta$  signals “backwards” by using integrators. Why might using numerical derivatives be bad? To simplify updating the model, define variables and use them as parameters in the Simulink blocks (e.g. instead of directly using the value of the pendulum length  $l$  in the block, define a variable  $l$  in Matlab and use it in the Simulink block). Include a figure of the Simulink model in your answer.
2. Plot the  $\theta$  response of this system to a single pulse having amplitude 10 and width 0.2 seconds. There are several ways to generate such a pulse; one easy way is to use a “pulse generator” with large period (e.g. 100s) and suitable pulse width (0.2% when using a period of 100s). When you plot the graph, show the response for 70 seconds of time. To what value is the pendulum angle converging to? *Hint:* At some point you should also plot your pulse input to verify that it is the correct amplitude and width. This does not need to be included in your lab report, but will help you avoid mistakes.
3. Plot the  $\theta$  response of this system to a pulse having amplitude 250 and width 0.2 seconds. To what new value does the pendulum angle converge? Explain why.
4. Replace the nonlinear term in your Simulink model with the linear version. Repeat the experiment for the pulse with amplitude 10 and width 0.2 seconds. Do the nonlinear and linear versions agree for the angle response?
5. Now try the linear version with the pulse having amplitude 250 and width 0.2 seconds. Do the nonlinear and linear versions agree for the angle response? Why is there a discrepancy?
6. Based on your results from parts 4 and 5, when is the linearized system a “good” approximation of the original system?