1. (15 pts) Equivalent Circuit (Nise 2.9)

Draw the equivalent electrical circuit for the system shown in Fig. 1.

2. (25 pts) Rotary Mechanical System (Nise 2.6, 2.7, 3.4, 3.5)

[10 pts] a. Find the transfer function for the system shown in Fig. 2, $\frac{\theta_2(s)}{T(s)}$.

[10 pts] b. Write the state space equations for this system in phase-variable form and find $A, B, C, D$.

[5 pts] c. Draw the equivalent block diagram of the system in phase variable form using integrator, scale, and summing blocks.

3. (10 pts) Linearization (Nise 2.11)

A system is described by $f(t) = m\ddot{x}(t) + b\dot{x}(t) + f_s(x, t)$, where $f_s$ is the force from a non-linear spring.

The spring force is defined by $f_s(x, t) = e^{100x} - 1$ where $x(t)$ is the spring displacement. Find the transfer function $X(s)/F(s)$ for small excursions around $x = 0$.

4. (15 pts) State Space (Nise 3.4, 3.5)

Given $Y(s)/U(s) = \frac{4s + 5}{s^4 + 5s^3 + 3s^2 + 5s + 13}$, write the state space equations for this system in phase-variable form and find $A, B, C, D$. (Hint: long division).

5. (15 pts) Transfer function from state space (Nise 3.6)

Find the transfer function $Y(s)/U(s)$ for the following systems:

[5 pts] a. $\dot{x} = A\dot{x} + Bu = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u(t)$ and $y = [1 \ 0 \ 0]x$

[10 pts] b. $\dot{x} = A\dot{x} + Bu = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -8 & 2 & -3 \\ 0 & 5 & 3 \\ -3 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 8 \\ 3 \end{bmatrix} u(t)$ and $y = [1 \ 3 \ 2]x$

6. (20 pts) Linearization (Nise 3.7)

For the system:

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
\dot{x}_4
\end{bmatrix} = 
\begin{bmatrix}
\log_e x_2 \\
\cos(\pi(u + x_4)) \\
\sin(\pi(u + x_2)) \\
x_3^2 + x_1^2
\end{bmatrix}
$$

Linearize the system about $x_1 = x_2 = 1, x_3 = 2, x_4 = -\frac{1}{2}$, $u = 1$, and express in state space form:

$\dot{\delta x} = A\delta x + B\delta u$. 